DataMOCCA

DATA MOdel for Call Center Analysis

Volume 4.1

The Call Center of "US Bank"

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DataMOCCA

DATA MOdel for Call Center Analysis

The DataMOCCA Project is an initiative of researchers from the Technion—Israel Institute of Technology and The Wharton School—University of Pennsylvania. The mission of the project is to collect, pre-process, organize and analyze data from Telephone Call/Contact Centers. The raw data obtained are call-by-call records of at least one year's duration from active Call Centers. Among the goals of the project are the development and distribution of Call Center databases, using a uniform schema. The data repository created, together with software tools, will be accessible through the world-wide-web and provide a resource for researchers and teachers of Service Engineering, Science and Management.

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1	Model Description and Introduction to User Interface	July 29, 2006
2	Summary Tables Variable Definitions	August, 2006
3.1	SEEStat Guide I – Beginning User	to be completed
3.2	SEEStat Guide II – Advanced User	July, 2008
3.3	SEEStat Guide III – Data Extraction Facility	to be completed
4.1	The Call Center of a "US Bank"	November 2, 2006
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1 Introduction

The source of this data base is a large Call Center of a US bank. It has sites in New York, Pennsylvania, Rhode Island, and Massachusetts (see Figure 4). The Call Center processes up to 300,000 calls a day, routes calls according to agent skills, and simultaneously queues calls across multiple sites. The Call Center provides the "correct" initial routing decision about 90 percent of the time and for the rest of the calls the center relies on a Network InterQueue. With the Network InterQueue, the Call Center routes the calls across a multi-node network based on business rules. The center provides several types of services; the most common are Retail, Premier, Business, Consumer Loans, Online Banking and Telesales.

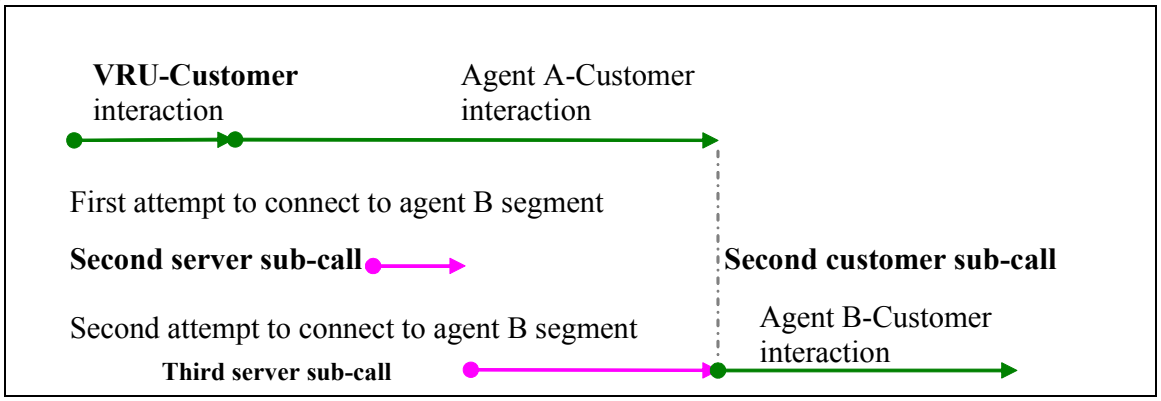
The Call Center consists of about 900-1200 agent positions on weekdays and 200-500 agent positions on weekends, unevenly distributed through the different nodes. These agents are service agents that represent the members of the primary agent group or super group. Working hours are 24 hours a day, 7 days a week. The data is compiled on a daily basis, from March 26, 2001 to October 26, 2003. There are 200,000–270,000 calls per weekday, 120,000-140,000 per Saturday and 60,000–100,000 calls per Sunday (based on April, 2001).

2 Data description

The database consists of ACCESS tables of daily data for the period from March 26, 2001 to October 26, 2003. A single call can consist of more than one segment; therefore it can occupy more than one record in the data table. A call segment record is constructed for each leg of the call. This record provides detailed information on the interaction between the customer and VRU, announcement, or agent; between two agents; and between the VRU and agent.

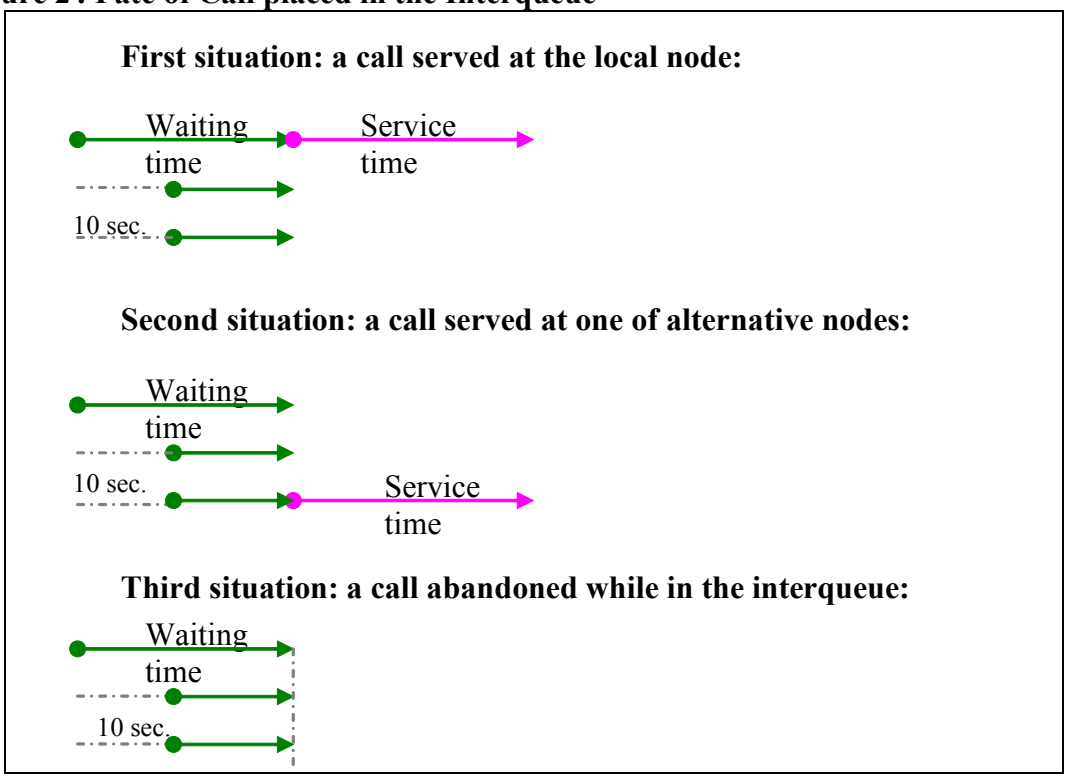
The call can consist of several customer sub-calls from the customer's perspective and several server sub-calls from server's perspective. Each customer sub-call includes the call segment records involving the customer interaction with a particular party. The second and subsequent customer sub-calls are: agent-initiated calls that represent customer interaction with the VRU; an Announcement; or service by another agent. The second and subsequent server sub-calls are the agent-initiated calls that occupy a new line in the system and continue in parallel or after the customer sub-call. For instance, if a customer dials into the Call Center and reaches the VRU, then s/he transfers to an agent, there would be a first customer and server sub-call; if thereafter the customer asks to speak with another agent and Agent A succeeds to connect the customer to Agent B at the second attempt, there would be two additional server sub-calls and a second customer sub-call.

Figure 1 : Customer and server sub-calls



As discussed earlier, the Call Center enables one to operate different sites or *nodes* from one real-time geographical location. Hence the definition of the nodes is technical. It could be that agents who are working in two nodes are located in the same place. Moreover, agents who are working on the same node could be geographically located in different places.

Figure 2 : Fate of Call placed in the Interqueue



All the nodes use the Call Center to route calls and to integrate agent desktops with customer databases. Pre-call routing provided by the Call Center network makes initial routing decisions, based on staffing at each site, and the Network InterQueue manages cross-node transfers. There are three situations that deal with tracking interqueued customer calls: either the call is served at the local node, namely the node where it originates, or served at one of the alternative nodes that can provide the desired service, or the call is abandoned. In these situations the call includes segments informing about the node where the call was served or abandoned, and the node/nodes from which the call is disconnected. Figure 2 demonstrates these three situations.

As a first step, for each month a MonthlyRecords table was produced. It includes information per day about missing days, the number of duplicate records (segments), the number of records with a UCID (a unique identifier for the call) that appears to be placed in the wrong day, the number of records that do not have any beginning of the call, the number of records with absent segments, the number of records with a new UCID given to the records with a different old UCID but with the identical Track (does not remain the same for segments that do not include the customer), the number of duplicated records after a new UCID. All these problematic calls were placed, separately from the intact calls, in a garbage Access file. Summary of the problematic calls over the period from March 26, 2001 to April 24, 2003 is shown in the following table:

Table 1 : Summary of Problematic Calls by Month

Month	missing days	duplicates	record_errors	local_missing	gaps	newUCID	duplicates_new UCID
March, 2001	25	4	2	0	0	0	0
April, 2001	0	1621	1454230	9	40	24151	43411
May, 2001	1	44337	224139	73	128	511	564
June, 2001	0	61118	16884	1038	0	1987	1987
July, 2001	0	23	1	4	19	0	7
August, 2001	0	7	419712	0	0	0	355
September, 2001	0	221912	659572	1015	302	131160	131930
October, 2001	0	8	40641	418	203	0	248
November, 2001	0	31358	221407	3	5	671	698
December, 2001	0	92949	405871	74	31	1728	1846
January, 2002	0	2	263263	11	43	0	46
February, 2002	0	8805	3	39	37	25	39
March, 2002	0	14708	679342	141	22	94	14762
April, 2002	0	675	108998	11	54	19892	20288
May, 2002	0	240191	593134	1	13	2439	2484
June, 2002	0	7	0	2	1	0	22
July, 2002	0	7	1865103	268	61	0	1267
August, 2002	0	6	9	0	4	0	101
September, 2002	3	1320	307524	3493	26	21	2475
October, 2002	0	4	3	6420	0	0	0
November, 2002	2	39164	160977	2454	2808	528	9124
December, 2002	0	0	0	0	14	0	3
January, 2003	1	43	6	104	44	0	95
February, 2003	0	193	126853	47	89	0	1473
March, 2003	0	175	6	0	7	0	3
April, 2003	10	50	35836	2	25	0	2688
Total	42	758687	7583516	15627	3976	183207	235916

In the second step, AppMap Access tables are produced and include descriptions of application numbers – based on the APP_# field in the original txt-files that were supplied separately. The specific application numbers are used to identify different agent groups (services). The meaning of the application numbers is different for

different switch nodes and periods of time; therefore AppMap-tables are produced accordingly for each switch node.

2.1 Summary Data

US Bank (from March 26, 2001 to October 26, 2003)

Table 2 : Summaries of Calls

	Total	Total Weekdays	Total Weekends	Average per Year	Average per Month	Average per Weekday
Total # of arriving calls	218,047,488	181,032,004	37,015,484	83,864,418	7,033,790	271,006
# Requesting agent service	41,646,142	37,036,994	4,609,148	16,017,747	1,343,424	55,445
# Served by IVR, Message or Announcement	176,401,346	143,995,010	32,406,336	67,846,672	5,690,366	215,561

Table 3 : Summaries of Agents

	Average per weekdays	Average per Saturdays	Average per Sundays
2001	1003	427	252
2002	872	332	154
2003	1145	365	146

Table 4 : Summary of Performance: weekdays

	IVR time	% Immediate Service	Average Queueing Time	% Abandoned	Average Service Time	% Continued Service
2001	1840216	77.1	31.9	0.9	250.3	13.4
2002	2521262	66.0	34.2	1.3	241.2	13.2
2003	2427314	65.2	33.3	1.1	248.4	13.3

Table 5 : Summary of Performance: Saturdays

	IVR time	% Immediate Service	Average Queueing Time	% Abandoned	Average Service Time	% Continued Service
2001	114953	72.6	38.7	1.8	236.6	9.3
2002	137643	63.4	46.4	2.1	227.7	9.4
2003	157163	60.9	46.8	1.9	237.5	9.4

Table 6 : Summary of Performance: Sundays

	IVR time	% Immediate Service	Average Queueing Time	% Abandoned	Average Service Time	% Continued Service
2001	82981	72.7	21.9	1.1	207.2	6.8
2002	92249	64.7	37.6	2.1	212.7	7.0
2003	104844	57.7	49.1	2.4	225.2	7.4

2.2 A typical day in April 2002

We have chosen a typical day – Tuesday, April 2, 2002 – since this day has virtually no problematic calls. There are a total of 261,143 calls. Figure 3 describes the process-flow of calls. There are 4 significant entry points to the system: through VRU ~ 87%, Announcement ~ 7%, Message ~ 2% and Direct group (callers that directly connect to an agent) ~ 1%; and there is an Others group ~ 3%, which includes the calls with undecipherable application numbers. About 79% of the calls exit from the system through the VRU, Announcement, Message and Others groups; while another 21% of callers entering these groups seek service by an agent. All callers that seek service by an agent, about 21% of incoming calls, form the Offered Volume. Note that a smaller proportion (13.6%) of overall VRU entries forms part of the Offered Volume.

At this stage, of the Offered Volume, agents handle 99% of the calls and 1% are abandoned. The served callers include those that will request other services by another agent (about 13% of the handled calls); with 87% of callers terminating the system after receiving service by a single agent.

Figure 3 : Daily flow of Total calls (Tuesday, April 2, 2002)

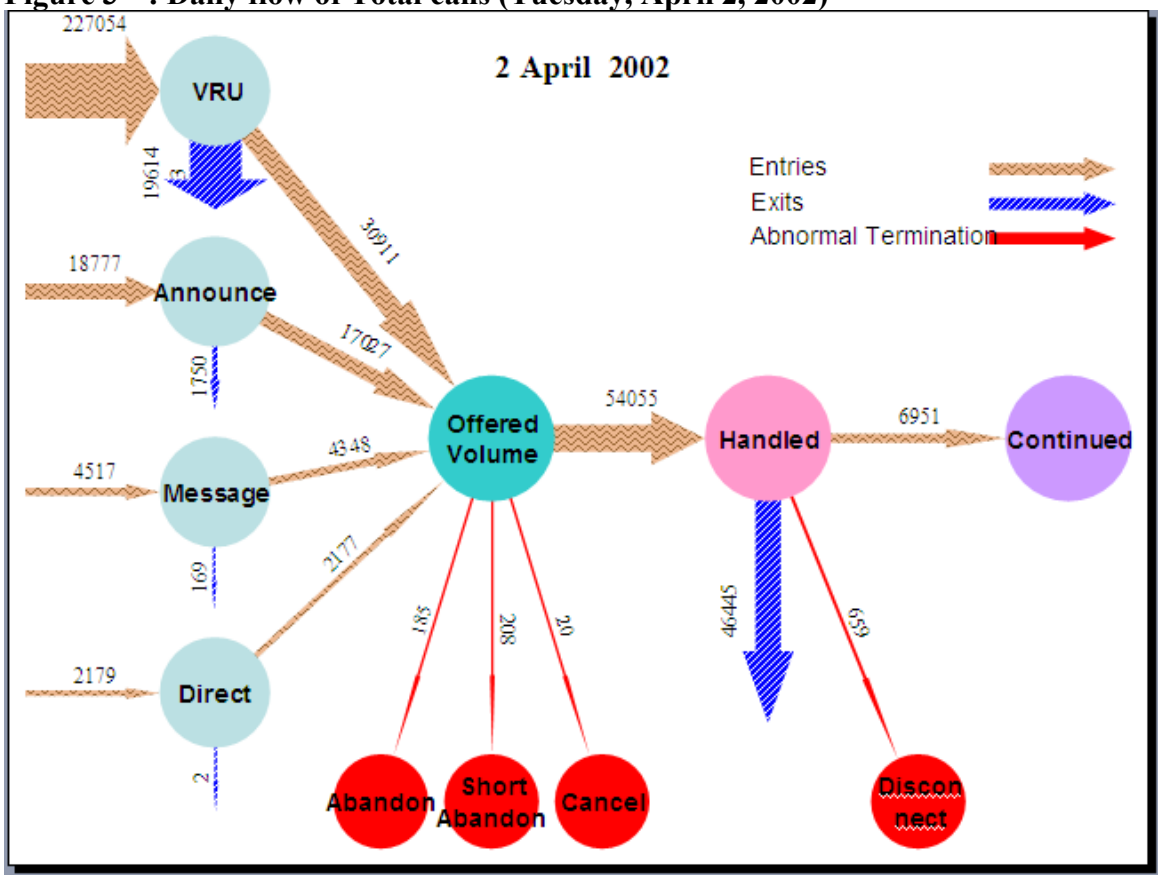


Table 7, Table 8 and Table 9 summarize the incoming calls according to type of services the call requests. The Retail, EBO and Spanish services are combined into one field.

Telephone Banking Performance Report by Line of Business
On Tuesday, April 2, 2002

Table 7 : Entries/Exits of System (% out of Total calls)

	Total	Retail EBO Spanish	Premier	Business	Platinum	Consumer Loans	Online Banking	Telesales
Total calls	261143	225554	5883	12996	268	4415	1943	3229
	227054	212638	3815	9910		691		
VRU	86.95%	94.27%	64.85%	76.25%		15.65%		
	18777	11681	1988	25	194	2774	1290	825
Announcement	7.19%	5.18%	33.79%	0.19%	72.39%	62.83%	66.39%	25.55%
	4517	25	52	2577		89		1774
Message	1.73%	0.01%	0.88%	19.83%		2.02%		54.94%
	2179	579	25	68		573	304	630
Direct	0.83%	0.26%	0.42%	0.52%		12.98%	15.65%	19.51%
From Other	1761	631	3	416	74	288	349	
Services	0.67%	0.28%	0.05%	3.20%	27.61%	6.52%	17.96	
	6855							
Other	2.62%							
	186109	173083	3386	9109		529		
VRU Exit	71.27%	76.74%	57.56%	70.09%		11.98%		
Announcement	5344	4333	451	66	2	321	100	70
Exit	2.05%	1.92%	7.67%	0.51%	0.75%	7.27%	5.15%	2.17%
	351	81	81	160		1		28
Message Exit	0.13%	0.04%	1.38%	1.23%		0.02%		0.87%
	13110	6155	2	70		7	1	28
Other Exit	5.02%	2.73%	0.03%	0.54%		0.16%	0.05%	0.87%
	1761	1422	78	229		27		
Other Services	0.67%	0.63%	1.33%	1.76%		0.61%		
	54468	40480	1885	3362	266	3530	1842	3103
Offered Volume	20.86%	17.95%	32.04%	25.87%	99.25%	79.95%	94.80%	96.10%

Table 8 : Handled/Unhandled calls (% out of Offered Volume)

	Total	Retail EBO Spanish	Premier	Business	Platinum	Consumer Loans	Online Banking	Telesales
	54055	40220	1873	3321	263	3500	1826	3052
Handled	99.24%	99.36%	99.36%	98.78%	98.87%	99.15%	99.13%	98.36%
Short	208	129	5	36	3	10	5	20
Abandon	0.38%	0.32%	0.27%	1.07%	1.13%	0.28%	0.27%	0.64%
	185	119	4	5		16	11	30
Abandon	0.34%	0.29%	0.21%	0.15%		0.45%	0.60%	0.97%
Other	20	12	3			4		1
Unhandled	0.04%	0.03%	0.16%			0.11%		0.03%
Offered Volume	54468	40480	1885	3362	266	3530	1842	3103

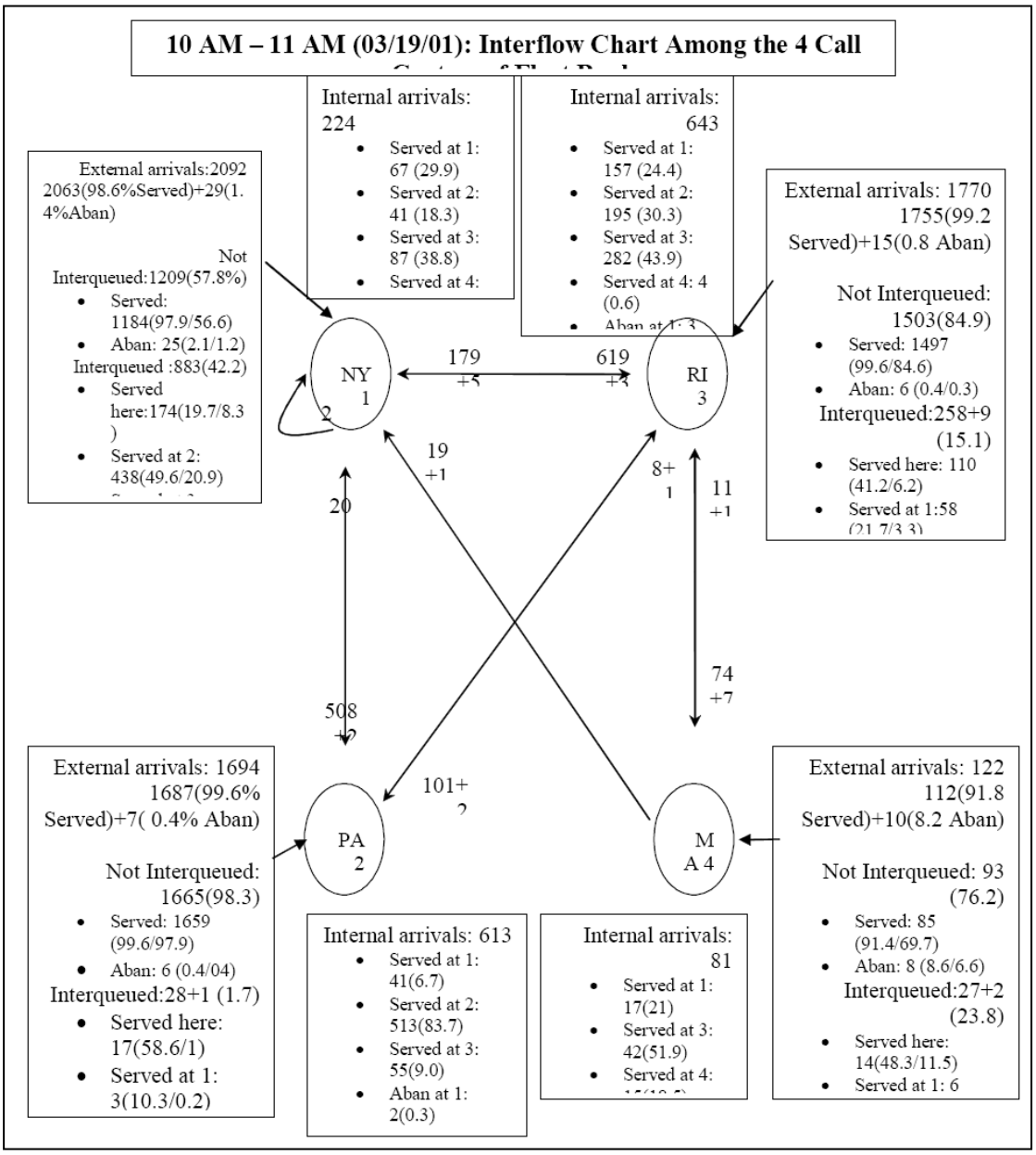
Table 9 : Abandon rates, non-waiting percentages and average waiting and service time

	Total	Retail	Premier	Business	Platinum	Consumer Loans	Online Banking	EBO	Telesales	Spanish
VRU	227054	212063	3815	9910		691				575
Percent of Nonwaiting	84.5	86.9	25.4	94.2	87.2	77.5	94.1	52.6	87.3	74.3
Average Waiting Time for All	3.2	3	3.9	1.6	2.3	3.1	2.5	7.5	6.1	23.1
Average Waiting Time for Waiting	17.4	18.4	5	16	13.4	11.7	34	15.2	46.9	87.6
Abandon Rate	0.8	0.6	0.6	1.2	1.1	0.8	0.9	0.2	1.6	2.7
Average Service Time	235.3	214.9	271.4	209.2	213.5	246.1	362.9	460.8	374.1	352.6

2.3 Network balancing via InterQueue

We have chosen one morning hour (between 10am and 11am) on a typical day – Monday, March 19, 2001. Figure 4 describes the process-flow of calls via the InterQueue. As discussed above, the Call Center has four sites: in New York (NY), Pennsylvania (PA), Rhode Island (RI) and Massachusetts (M). We observe that NY is most overloaded node (2,092 external arrivals), and M gets only 122 external arrivals. Note also that most of the interqueued calls are from NY node: 619 to RI and 519 to PA. The percent of the interqueued calls is the lowest at PA node: only 28 calls out from 1,694 are switched to other nodes.

Figure 4 : Network balancing via InterQueue (Monday, March 19, 2001)



3 Case studies

First, we demonstrate how DataMOCCA can be used to identify operational roots of congestion that cause deterioration in service levels. We envision such applications as supporting daily root-cause analysis, which is a common requirement in many call centers. Then we study the influence of network balancing protocol on the performance level. Finally, we consider some interesting distribution for different customer types.

3.1 Sample analysis of peak Telesales abandonment, October 2001

- Figure 5 identifies a peak in the **abandonment** Telesales calls in October 2001.
- Focusing on that month, we see unusually large values of **abandonment** on Tuesday, October 9 and Wednesday, October 10 (Figure 6); for example, over 50% on October 10.
- A monthly picture identifies large values in the Telesales calls **arrivals** on October 9-10 (Figure 7). The explanation of this is that October 9 is the first day after three weekends (Saturday, Sunday and Columbus day).
- As shown in Figure 8, the number of agents in the period October 9 – October 11 is larger than for comparable days in October.
- Focusing on October 10, we see that nearly 50% of the calls between 9:00 and 22:00 actually abandoned (see Figure 9).
- Hence, the somewhat increased number of agents on October 9-10 is insufficient for sustaining the usual service level.

Figure 5 : Abandonment rate of Telesales calls

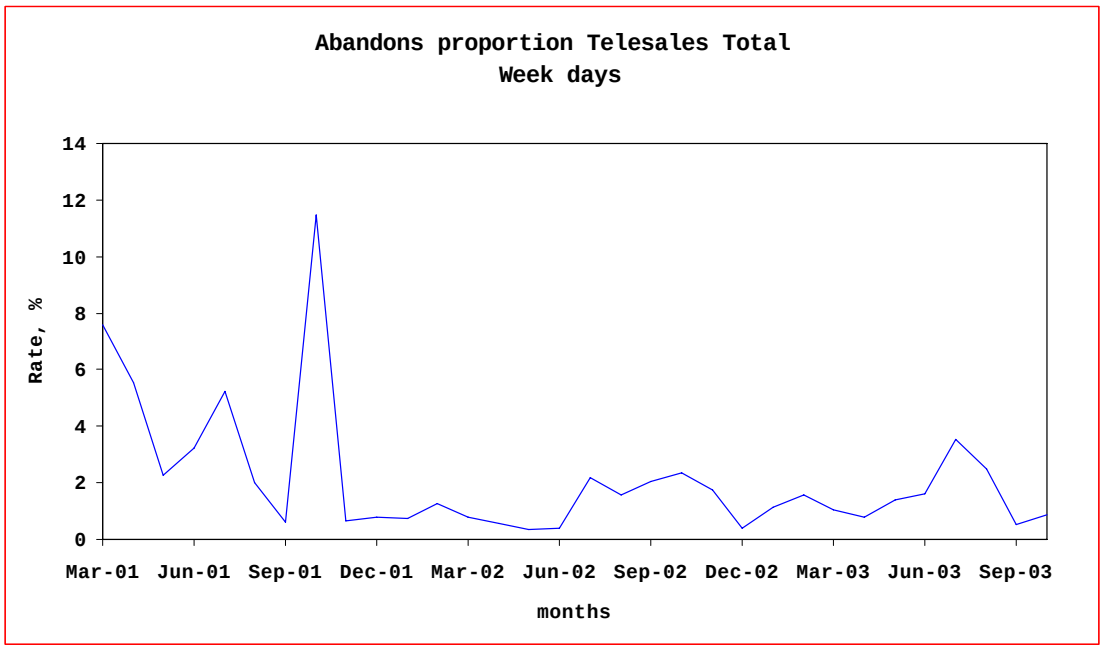


Figure 6 : Abandonment rate of Telesales calls (Oct-01)

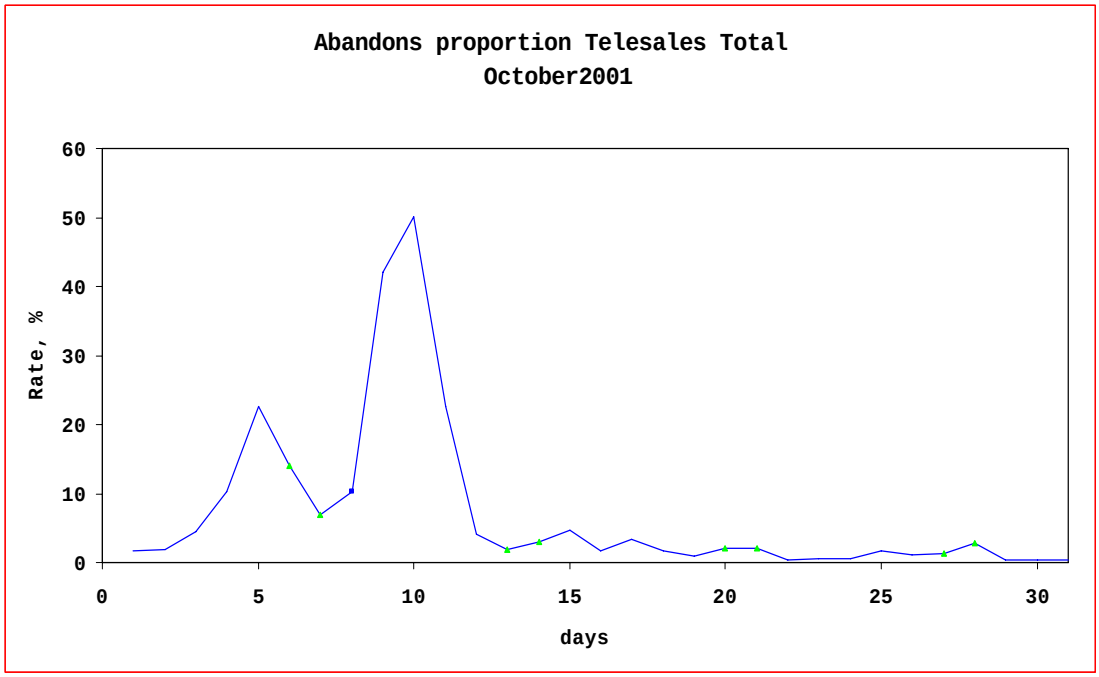


Figure 7 : Arrival rates of Telesales calls (Oct-01)

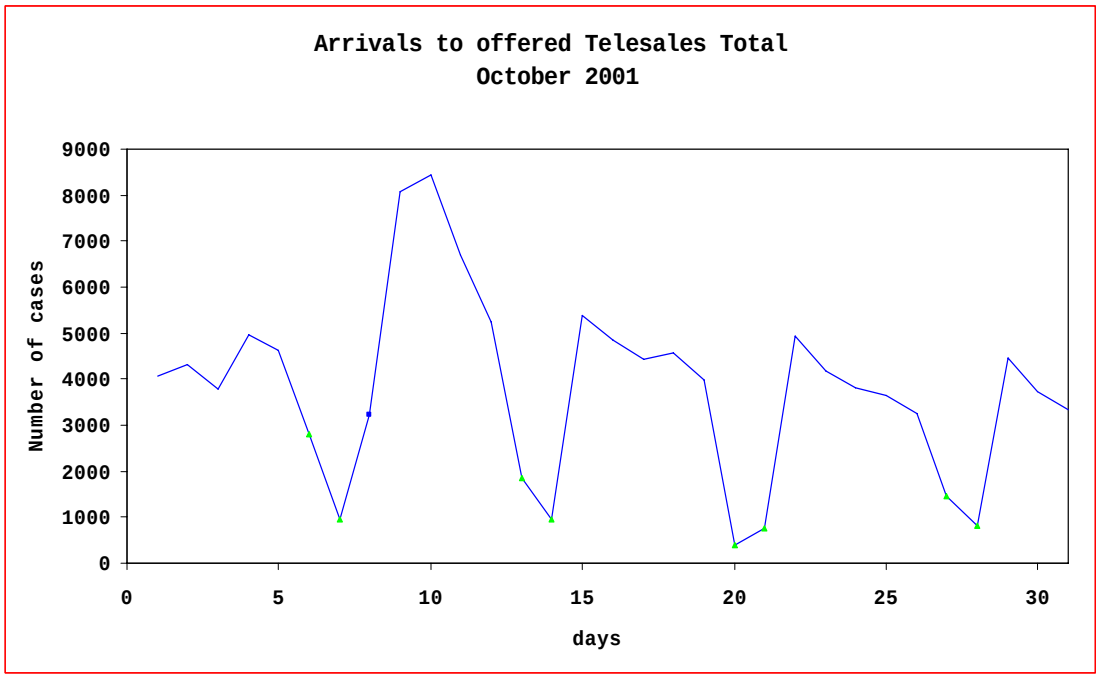


Figure 8 : Number of agents for Telesales calls (Oct-01)

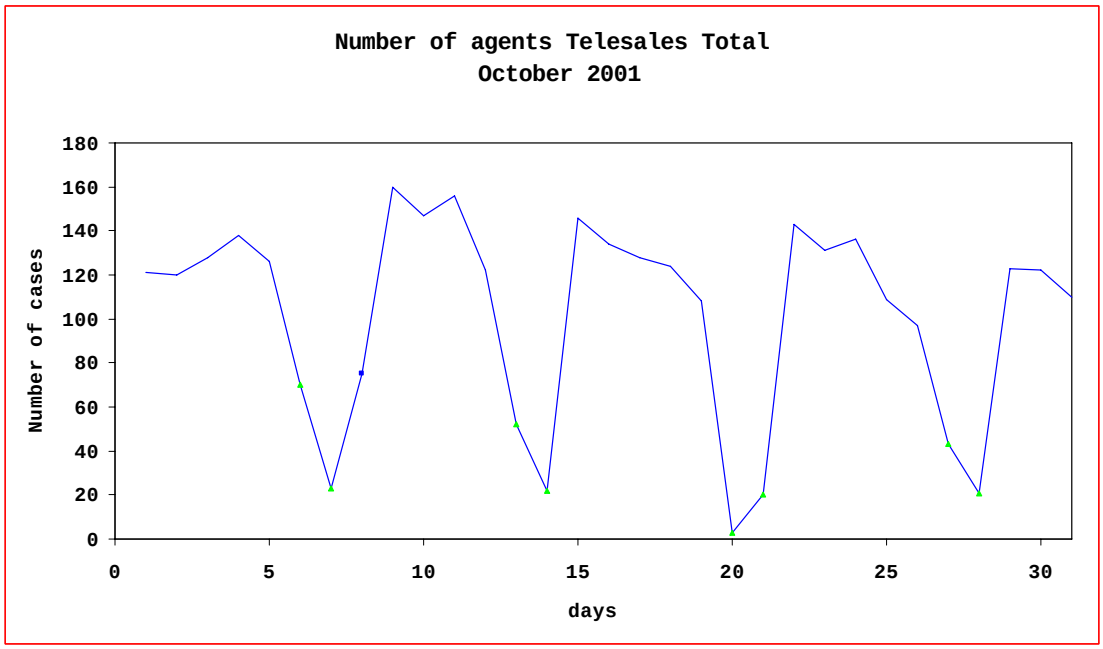
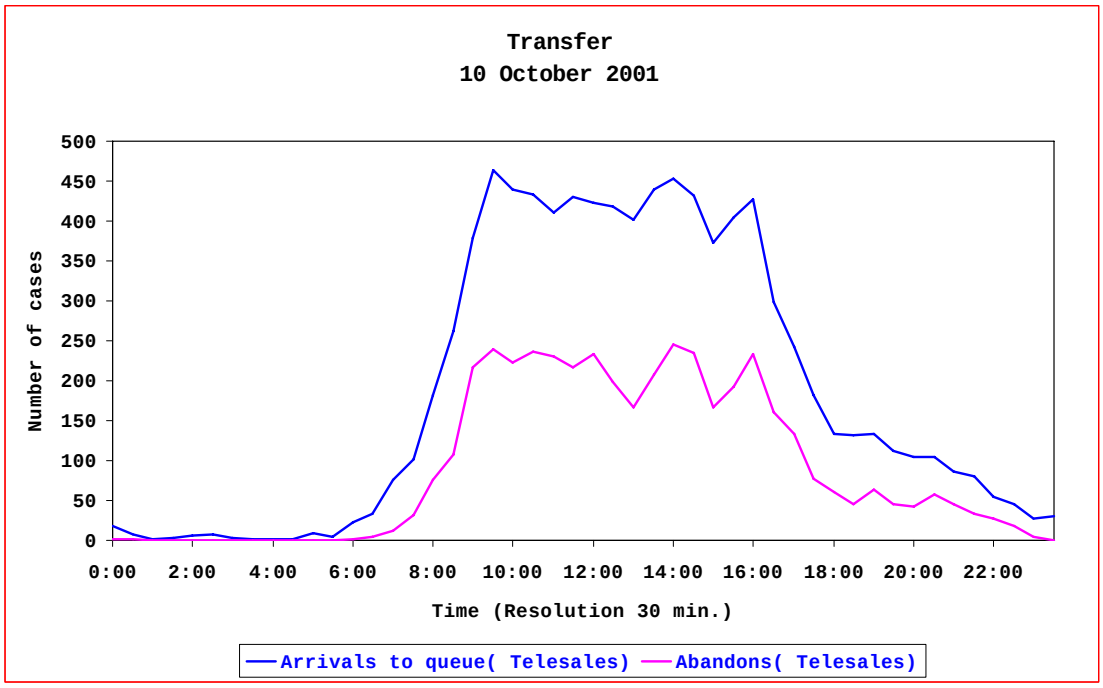


Figure 9 : Arrivals and abandonment rate of Telesales calls (Wednesday, October 10, 2001)



3.2 Sample analysis of peak Telesales unhandled, May 2003

- A monthly picture identifies a peak in the **unhandled** Telesales calls, on Wednesday, May, 28, 2003 (see Figure 10).
- Focusing on that day, we see a peak of **unhandled** calls in 13:00, while a significant decrease of the arrival rate took place (see. Figure 11).

- Using Agent Status of Telesales, we observe that there was a significant decrease of agents serving incoming calls between 12:00 and 13:00 (from 57 to 31) and sharp increase in the number of agents who were on a long-break (see Figure 12).

Figure 10 : Number of unhandled for Telesales calls (May-03)

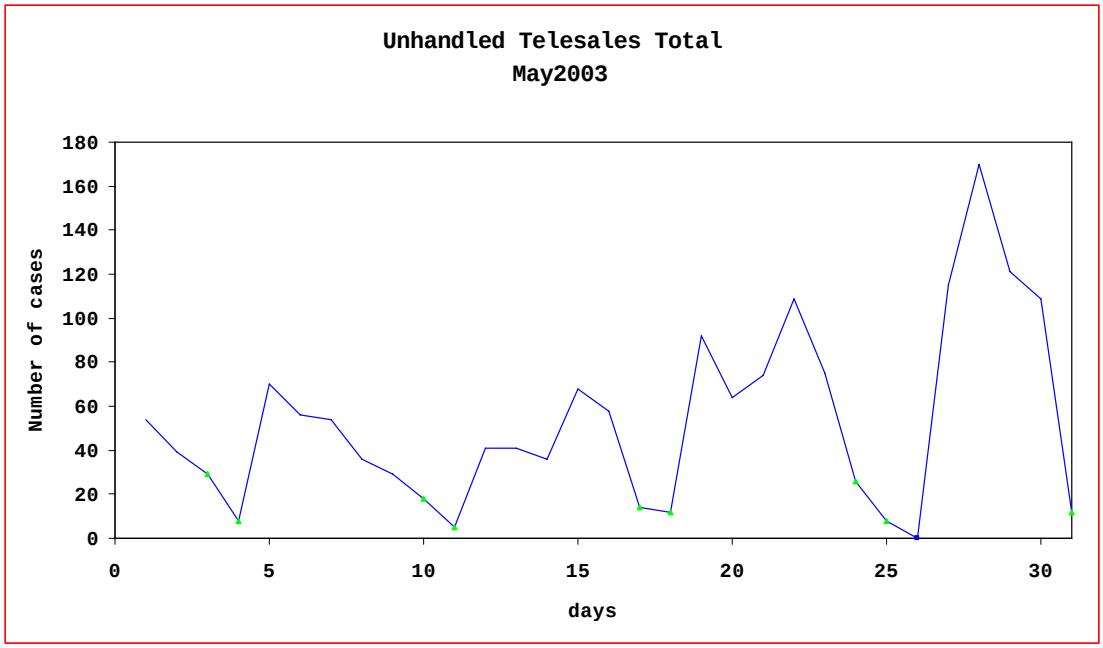


Figure 11 : Number of arrivals and unhandled calls for Telesales (Wednesday, May 28, 2003)

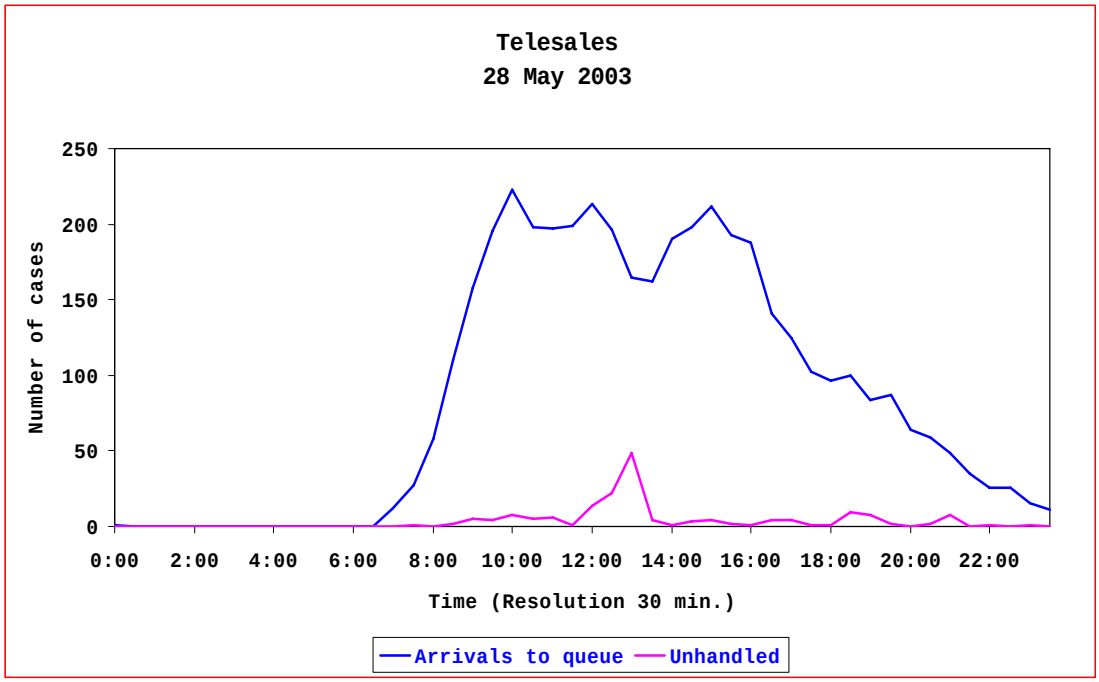
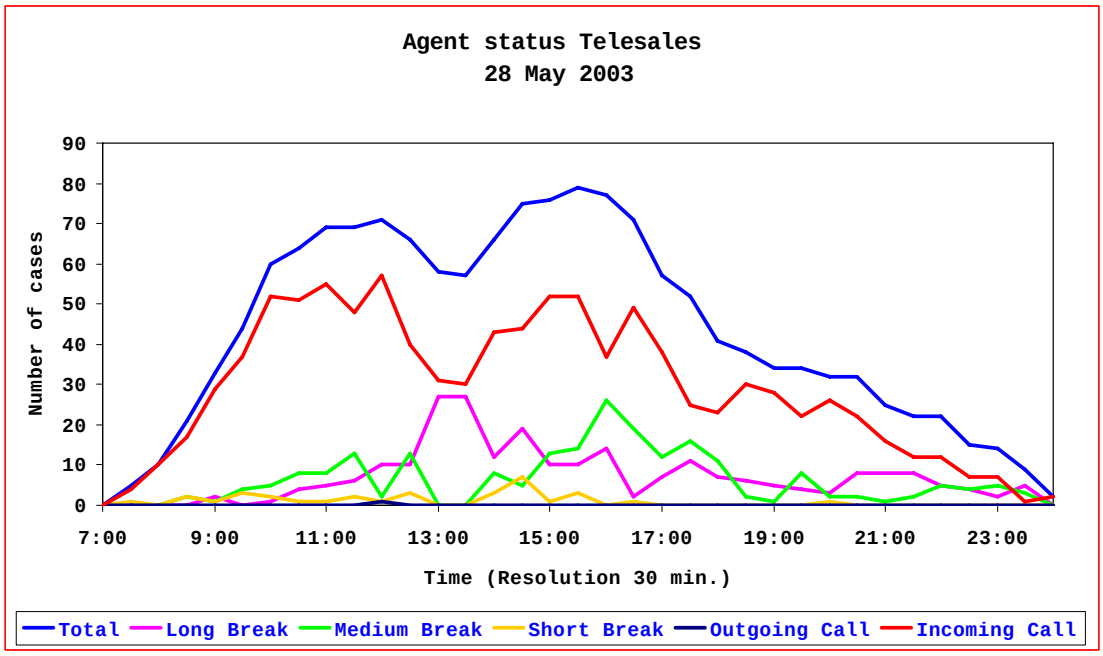


Figure 12 : Agent status for Telesales calls (Wednesday, May 28, 2003)



3.3 Sample analysis of peak Consumer Loans waiting time

- Figure 13 identifies a peak in the average waiting time of Consumer Loans calls in October 2001.
- Focusing on that month, we see two periods with unusually large values: on October 9-11 and on October 22-23 (see Figure 14).
- We study in detail two days: Wednesday, October 10 and Thursday, October 11. Focusing on October 10, we see maximal waiting time between 9:00 and 11:30am (Figure 15).
- We compare now all Wednesdays in October with respect to average number of arrivals and number of agents. As shown in Figure 16, the maximal number of arrivals took place on October 10.
- However, the difference between numbers of agents for Wednesdays compared to October 10 between 9:00 and 10:30 is relatively small (Figure 17).
- Focusing now on October 11, we see three periods with relatively large waiting time: between 8:30 and 10:30, between 14:30 and 16:00 and between 18:30 and 19:00 (Figure 18).
- We compare now all Thursdays in October with respect to average number of arrivals and number of agents. We observe a maximal arrival volume on October 11 (Figure 19).
- However, we see no difference between number of agents on October 11 and on other Thursdays (Figure 20).
- Hence, the number of agents on October 10 and 11 is insufficient for sustaining the usual service level.

Figure 13 : Average waiting time for Consumer Loans calls

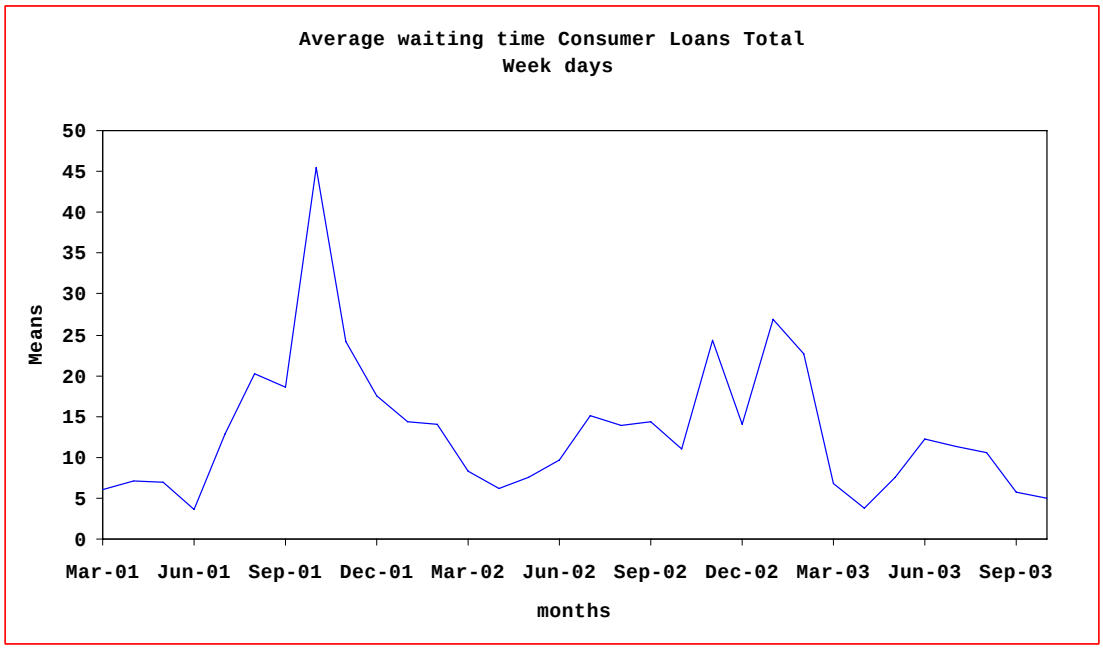


Figure 14 : Average waiting time for Consumer Loans calls (Oct-01)

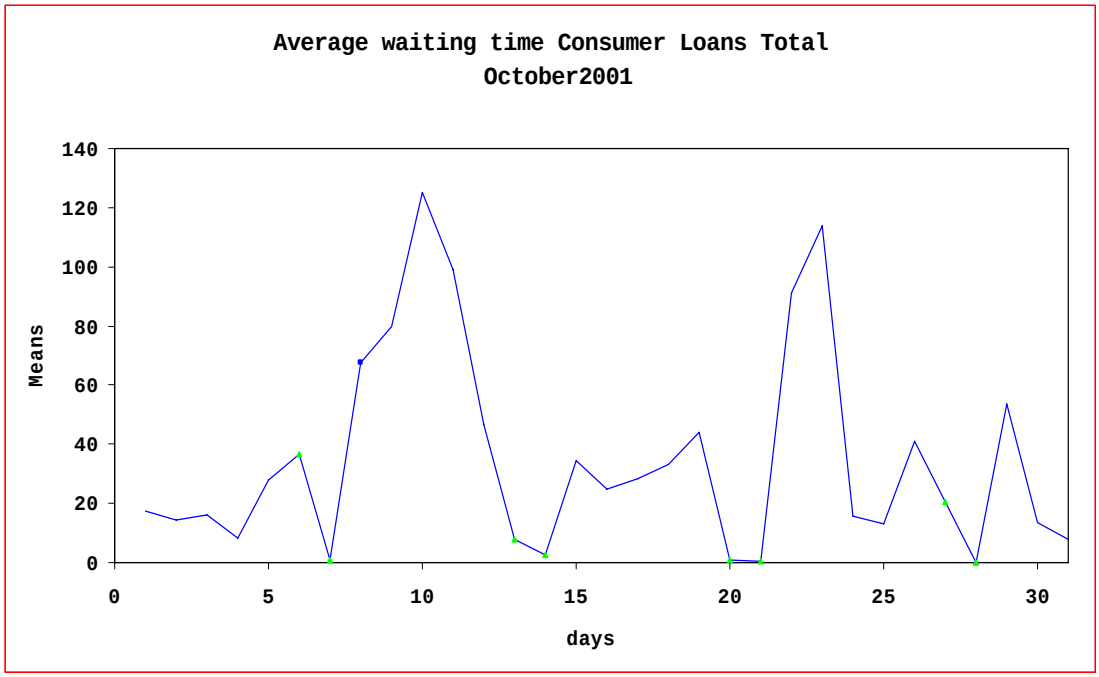


Figure 15 : Average waiting time for Consumer Loans calls (Wednesday, October 10, 2001)

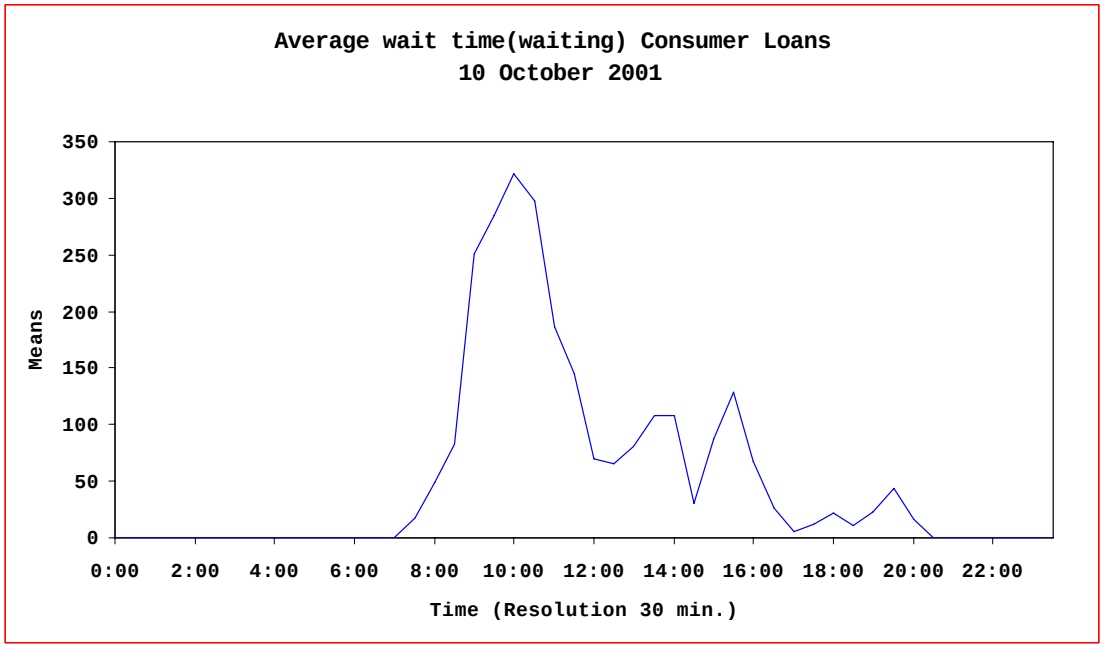


Figure 16 : Arrival rates of Consumer Loans calls (Wednesday, October 10, 2001)

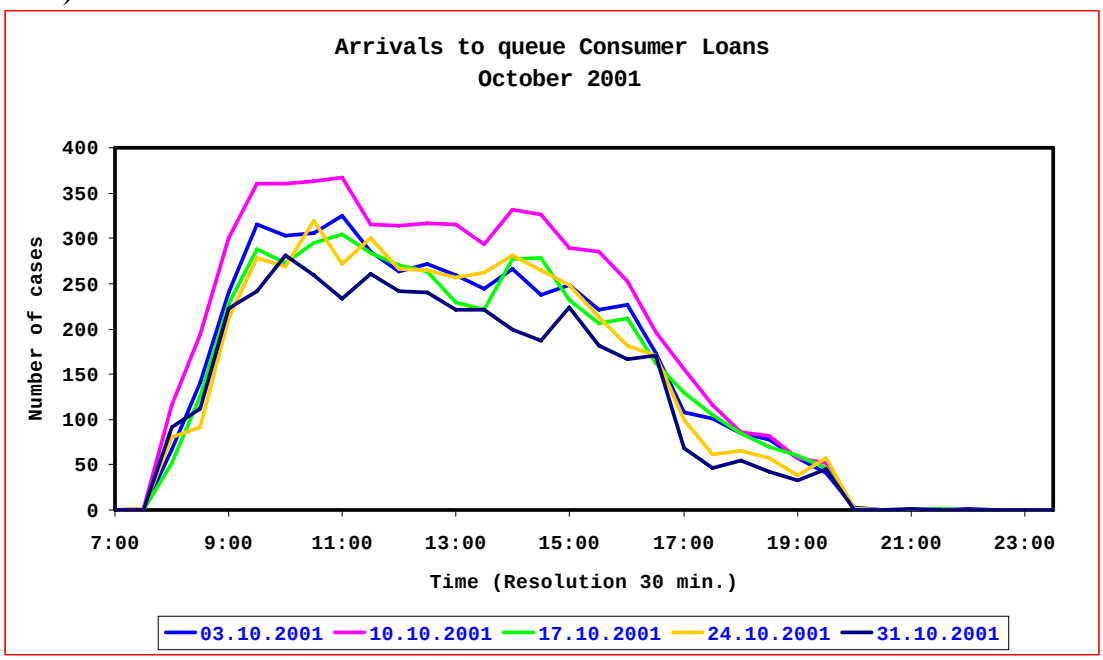


Figure 17 : Number of agents rate of Consumer Loans calls (Wednesday, October 10, 2001)

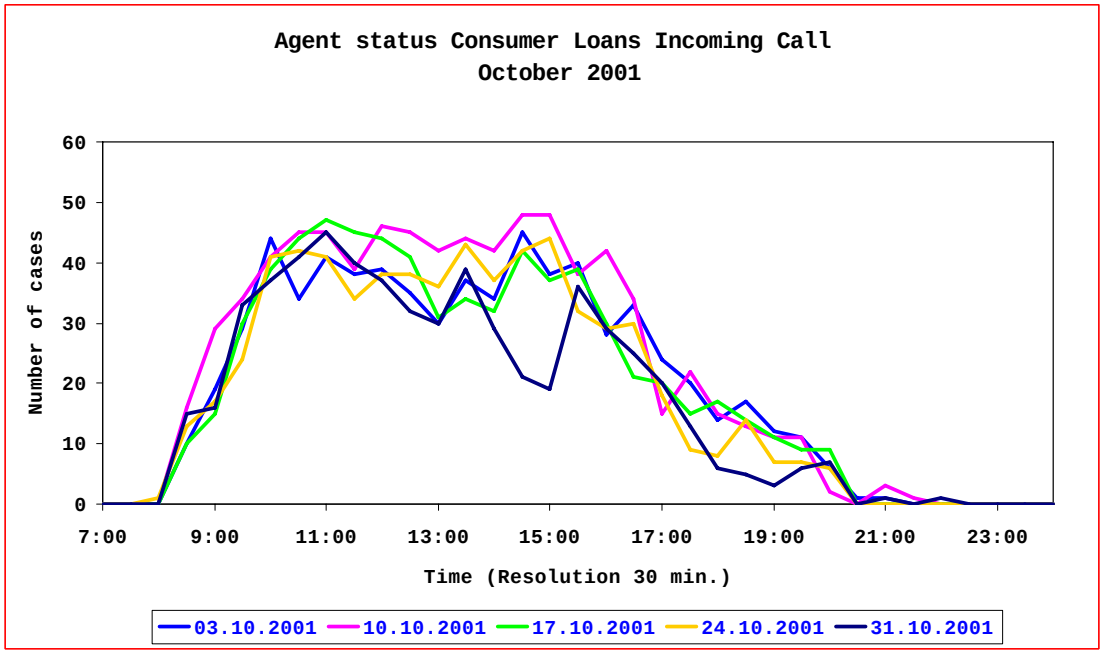


Figure 18 : Average waiting time for Consumer Loans calls (Thursday, October 11, 2001)

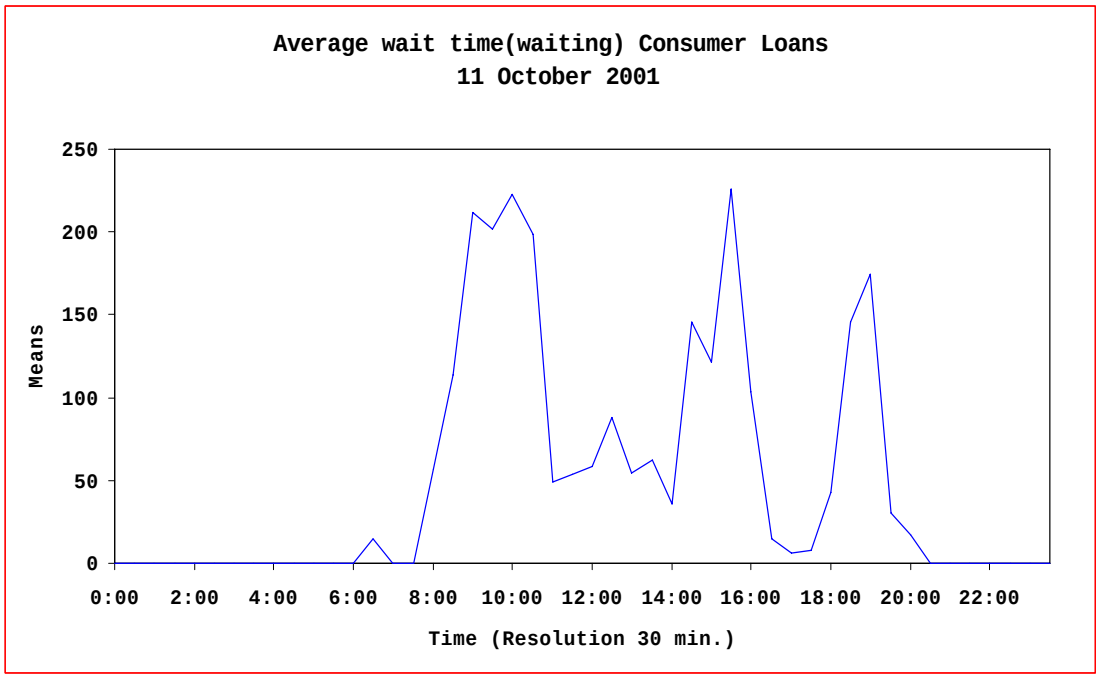


Figure 19 : Arrival rates of Consumer Loans calls, Thursday (Thursday, October 11, 2001)

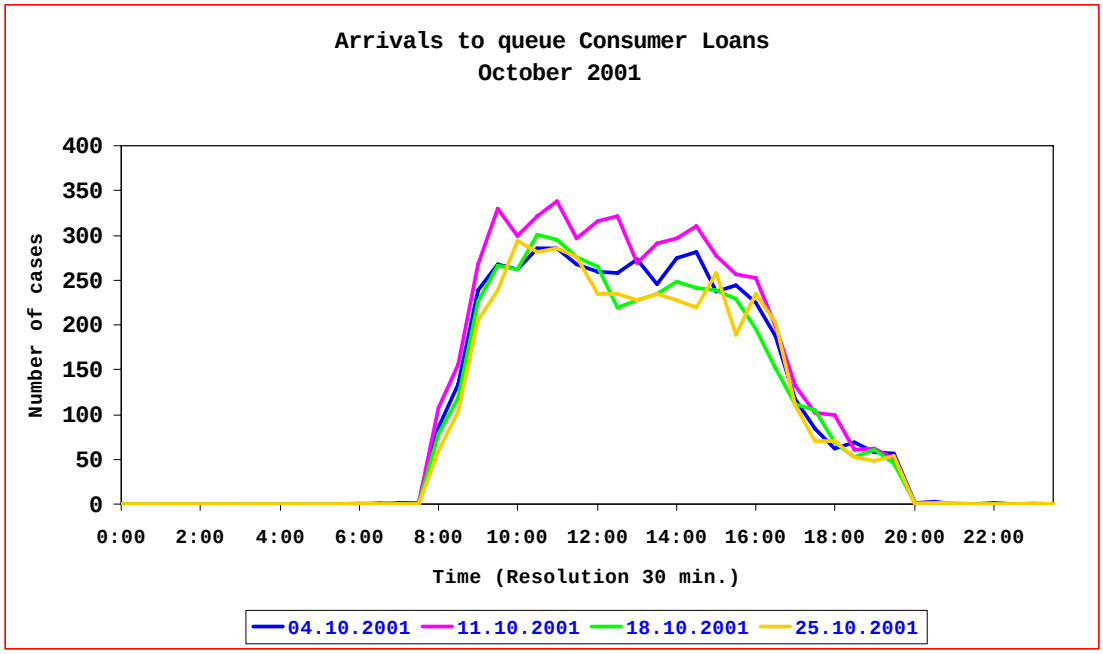
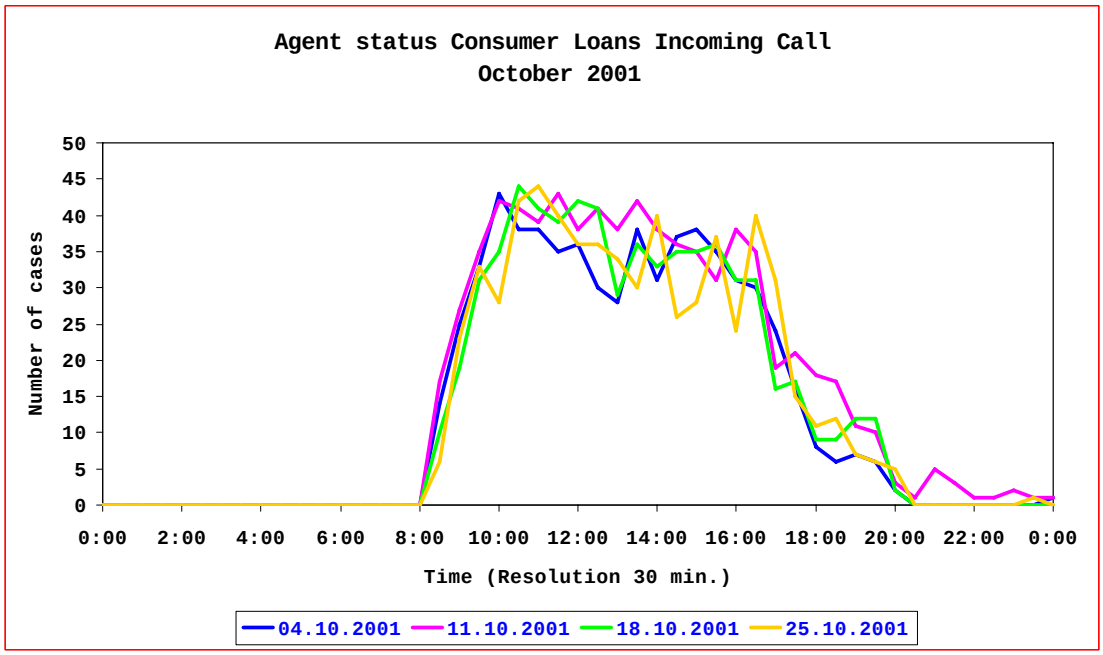


Figure 20 : Number of agents rate of Consumer Loans calls (Thursday, October 11, 2001)

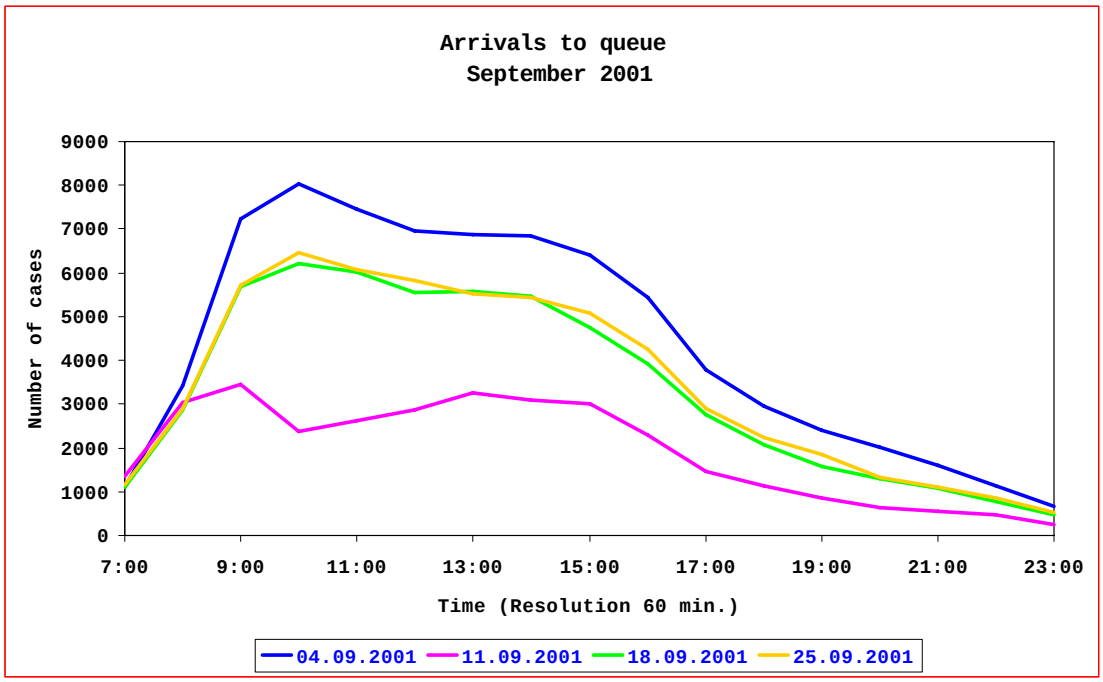


3.4 Sample analysis of arrival rate, September 11, 2001

Figure 21 presents an hourly call volume during Tuesdays in September, 2001. We observe that the curves almost coincide for September 18 and September 25. Note that average number of calls on September 4 is larger that the other Tuesdays. The explanation of this is that September 4 is the first day after a holiday (Labor Day).

Finally, we observe a significant decrease of calls after 8am on September 11, which, of course, is expected.

Figure 21 : Average number of calls per hour (Sept-01, Tuesdays)



3.5 Sample analysis of agent status for Business calls, June 4, 2001

Figure 22 presents the distribution of different types of agents per hour for Business calls on Monday, June 4, 2001. We observe a non-effective choice of number of agents: more than half of the agents are on break during all day, except 2-3:30pm. Indeed, during most of the day, a relatively small percentage of the total number of agents is busy serving incoming calls (for example, 27-26 out of 68-74 between 11-11:30am). However, we observe that it is no affect to service level. Figure 23 and Figure 24 present unhandled proportion and average waiting time for Business calls during Mondays in June 2001. Note that the June 4 does not differ from other Mondays.

Figure 22 : Agent status for Business calls (Monday, June 4, 2001)

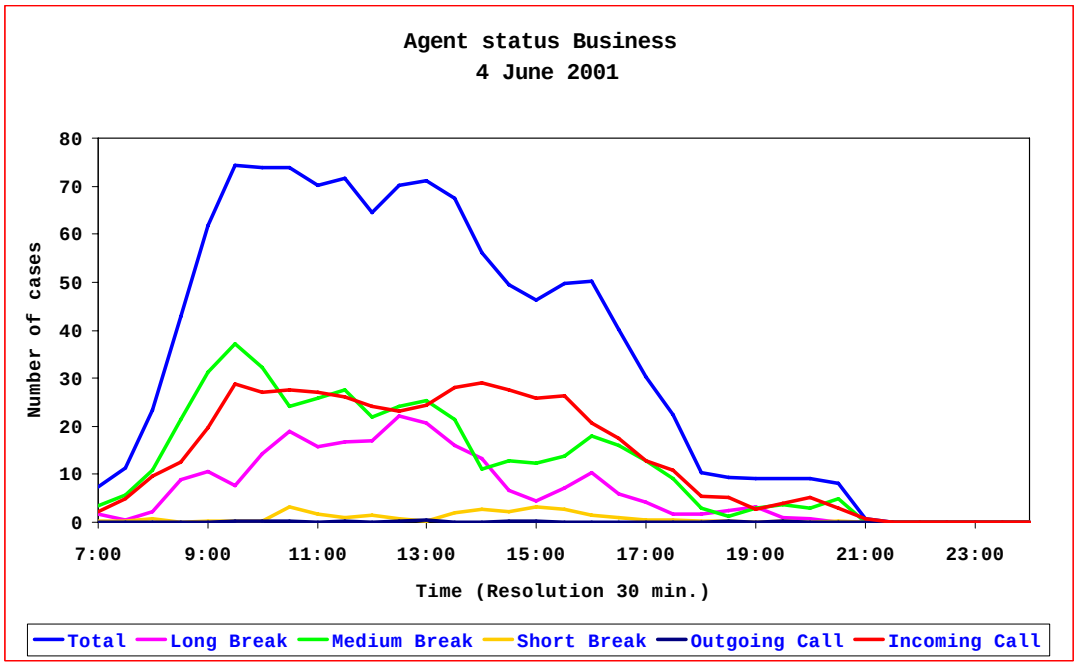


Figure 23 : Unhandled proportion for Business calls per hour (Jun-01, Mondays)

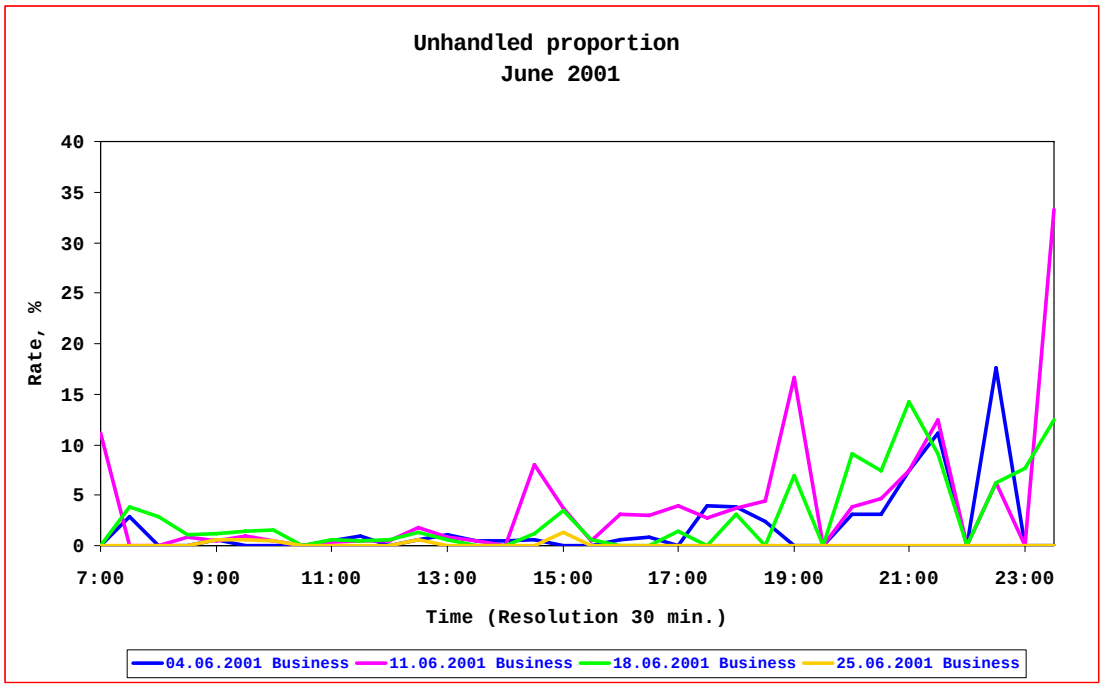
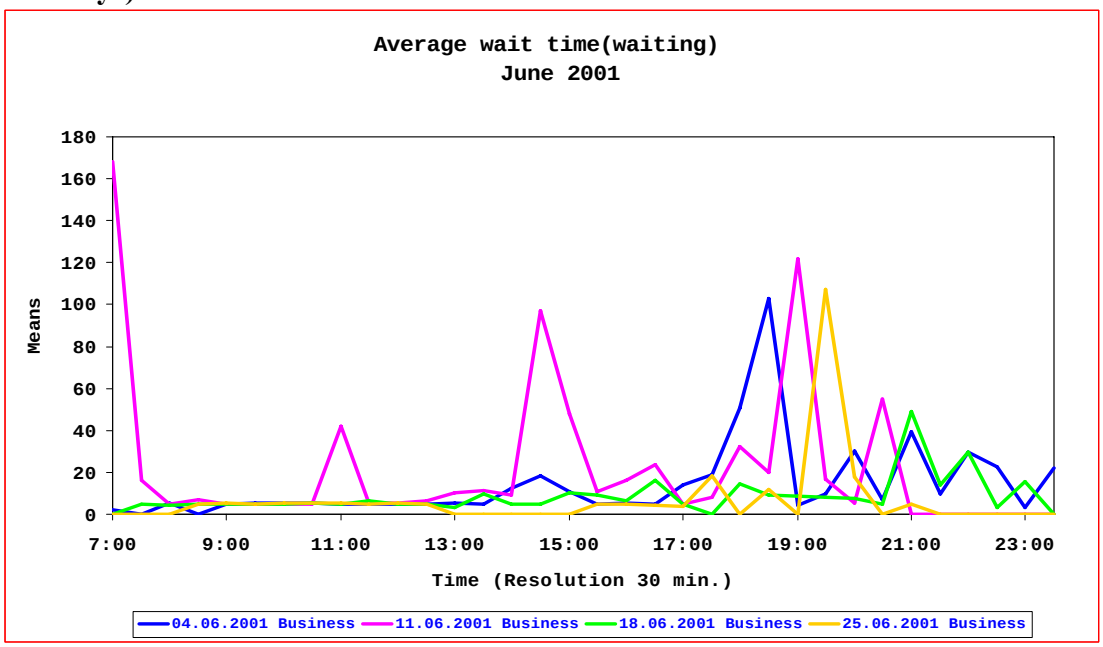


Figure 24 : Average waiting time (waiting) for Business calls per hour (Jun-01, Mondays)



3.6 Service without waiting for Regular vs. High-Priority customers

Figure 25 and Figure 26 present rates of Retail and Premier customers (Business and Platinum, respectively) who received service immediately (customers who waited 0 or 1 second). We see that Premier and Platinum customers do not get a better service level, in spite of Premier is the high priority version of Retail, and Platinum is the high priority version of Business. The story will be continued in Subsection 7.2.

Figure 25 : Rate of Retail and Premier calls served without waiting (Tuesday, October 8, 2002)

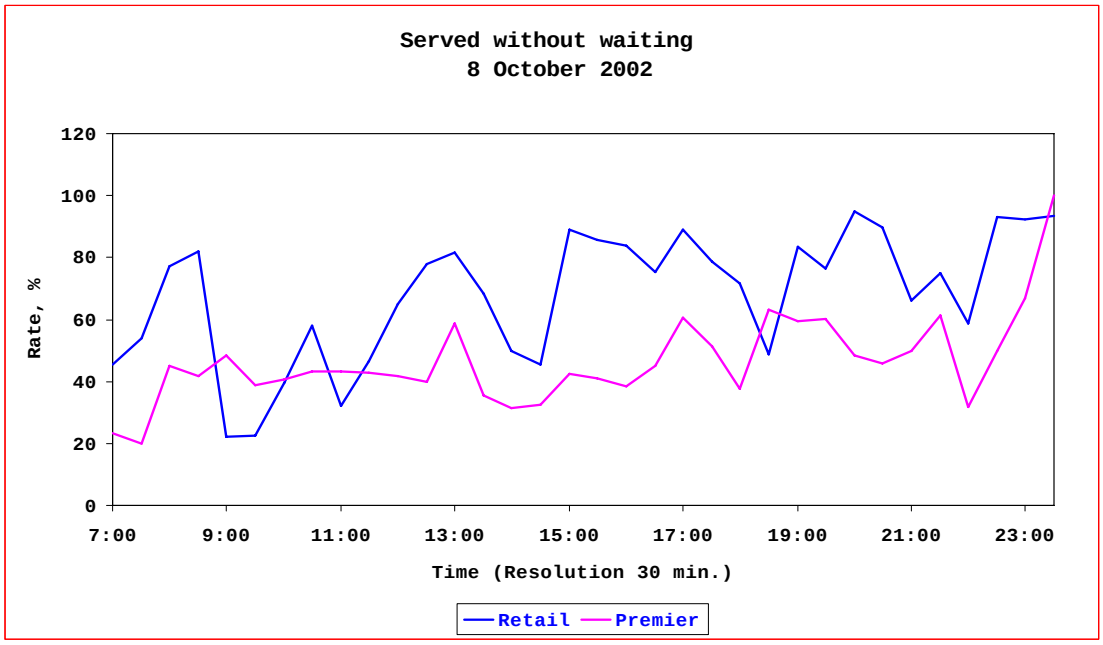
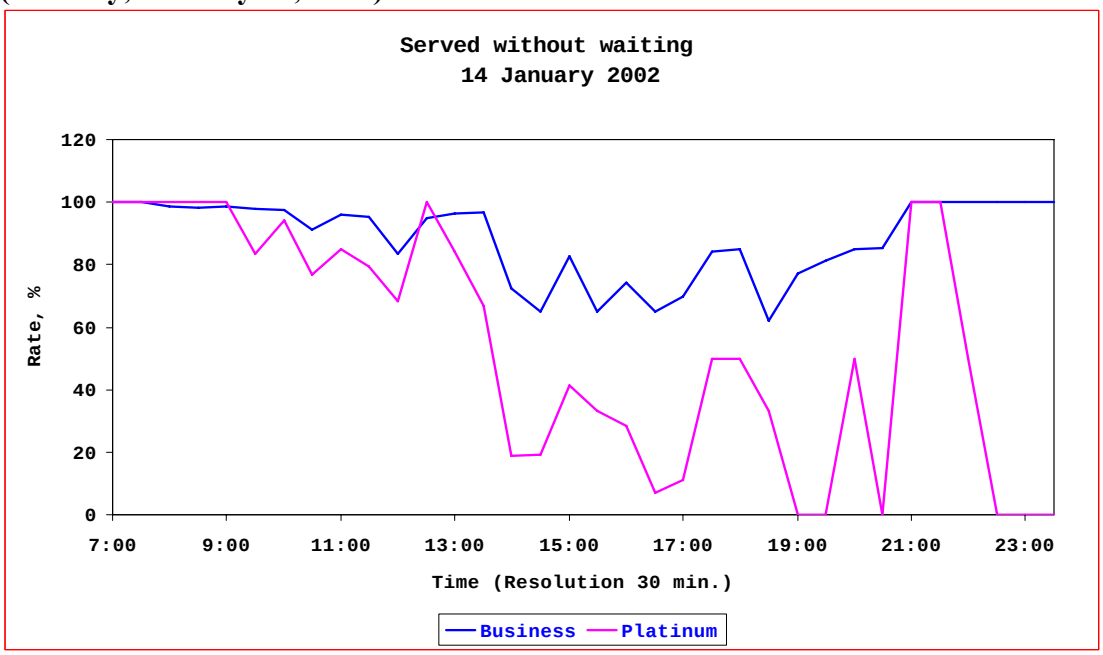


Figure 26 : Rate of Business and Platinum calls served without waiting (Monday, January 14, 2002)



3.7 Queues and Staffing for Retail calls, May 3, 2002

Figure 27 demonstrates a staffing problem of Retail calls on May 3, 2002. We observe arrival peak at the beginning of the working day (blue line). Number of agents at 7-8am is small (violet line). It led to a peak of the queue length (green line) and to significant peaks of abandon proportion (about 20%; see Figure 28) and of average waiting time (see Figure 29).

Figure 27 : Queues and Staffing for Retail calls (Friday, May 3, 2002)

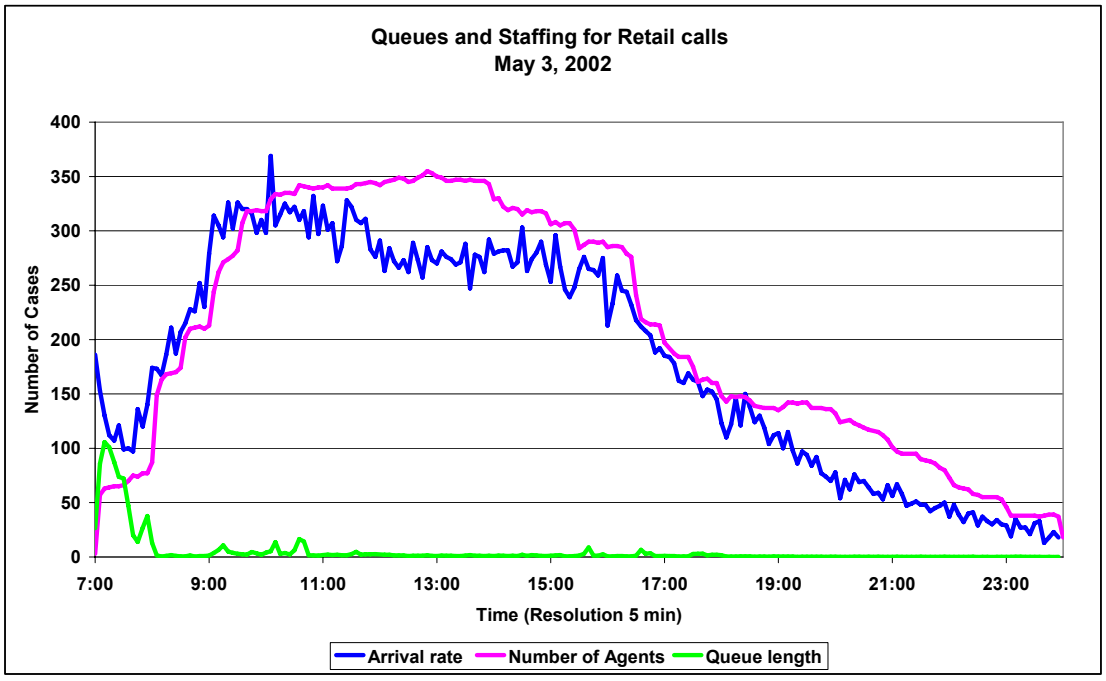


Figure 28 : Abandon proportion for Retail calls (Friday, May 3, 2002)

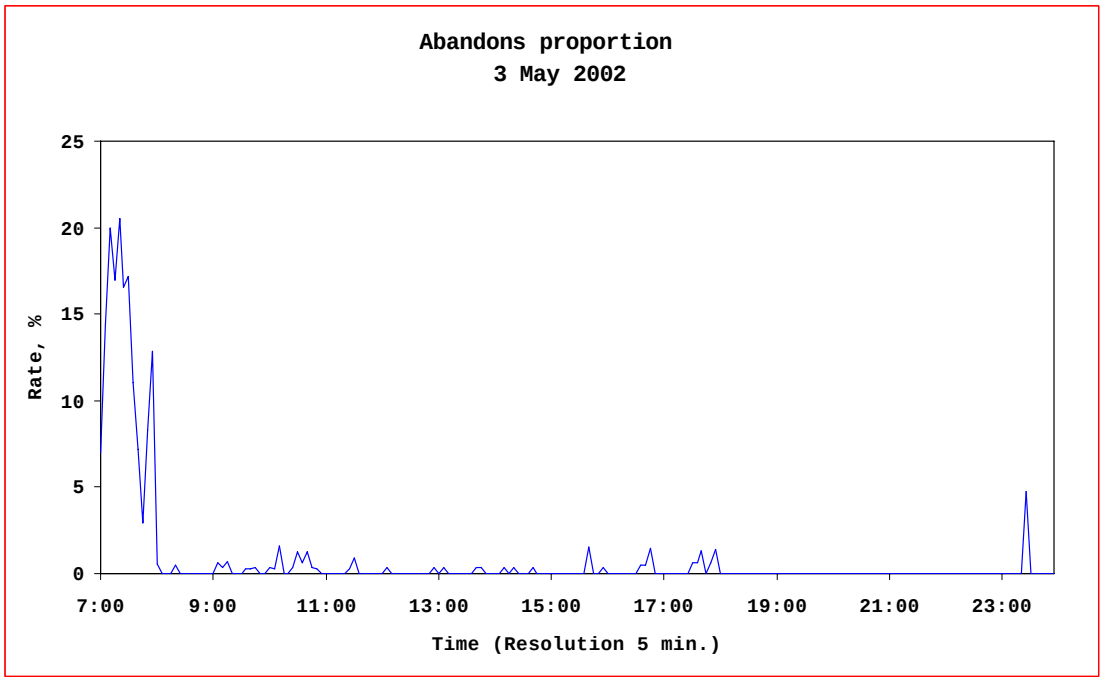
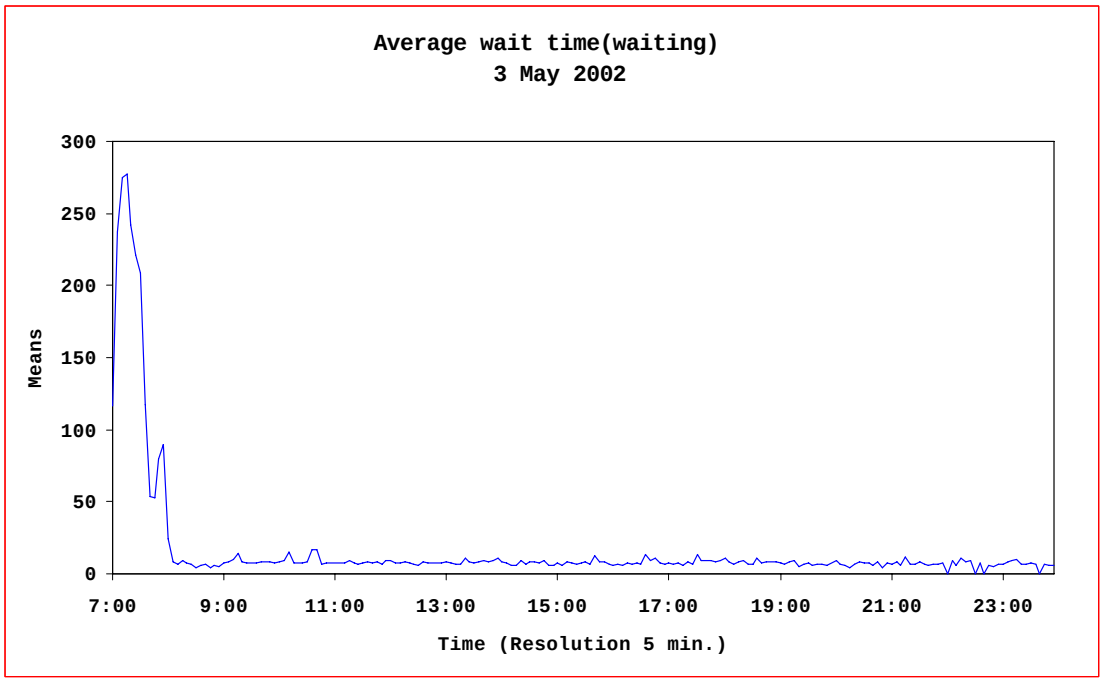


Figure 29 : Average waiting time (waiting) for Retail calls (Friday, May 3, 2002)



3.8 Network Balancing Protocol and Performance Level

Figure 30 and Figure 31 presents distributions of waiting time for Retail and Business calls in August 2001. We observe a peak for Retail on 10 sec. After 10 seconds of wait, Retail customers were sent into the interqueue. Histogram of waiting time for Business calls has a peak at 5 seconds for the same reason. The second peak at 18 seconds unclear: maybe a priority-upgrade took place.

Figure 30 : Histogram of waiting time for Retail calls (Aug-01)

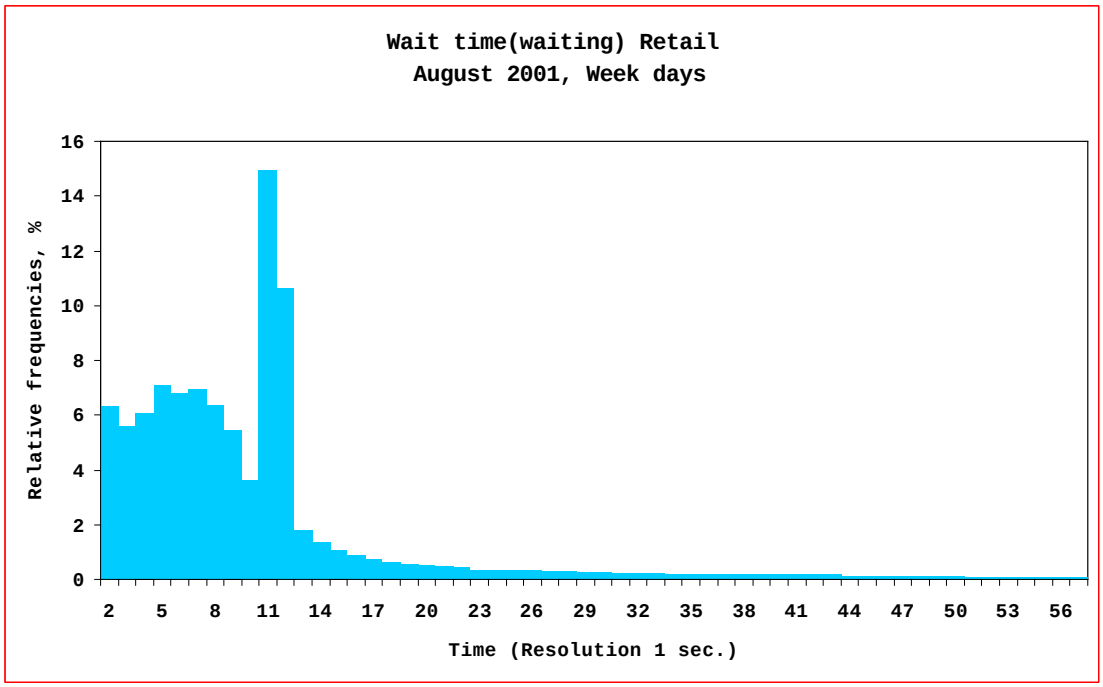
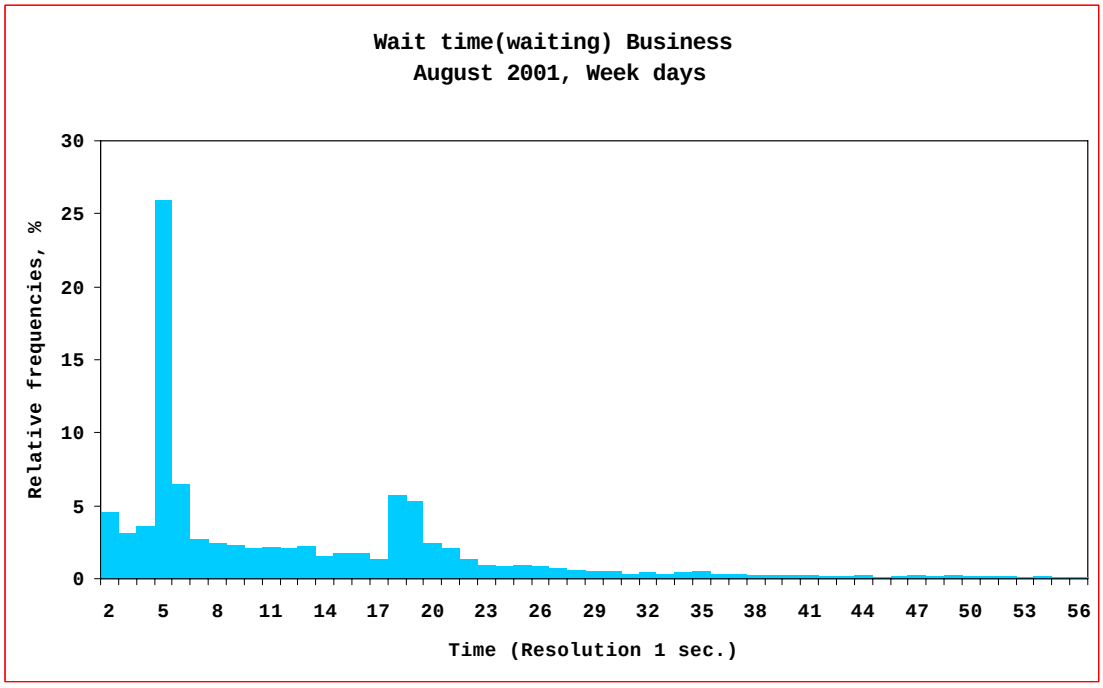


Figure 31 : Histogram of waiting time for Business calls (Aug-01)



3.9 Interesting Distributions

Now consider some interesting distribution for different customer types.

Figure 32 presents the distribution of agent service time for Retail calls in December 2002. The agent service time for Retail calls has a distribution that is similar to the lognormal distribution, except for a relatively large percent of small service times. These short service times probably reflect other phenomena – and not real service. In

some cases the customer (or even the agent!) may disconnect as soon as the connection is made.

Figure 32 : Distribution of agent service time for Retail calls (Dec-02)

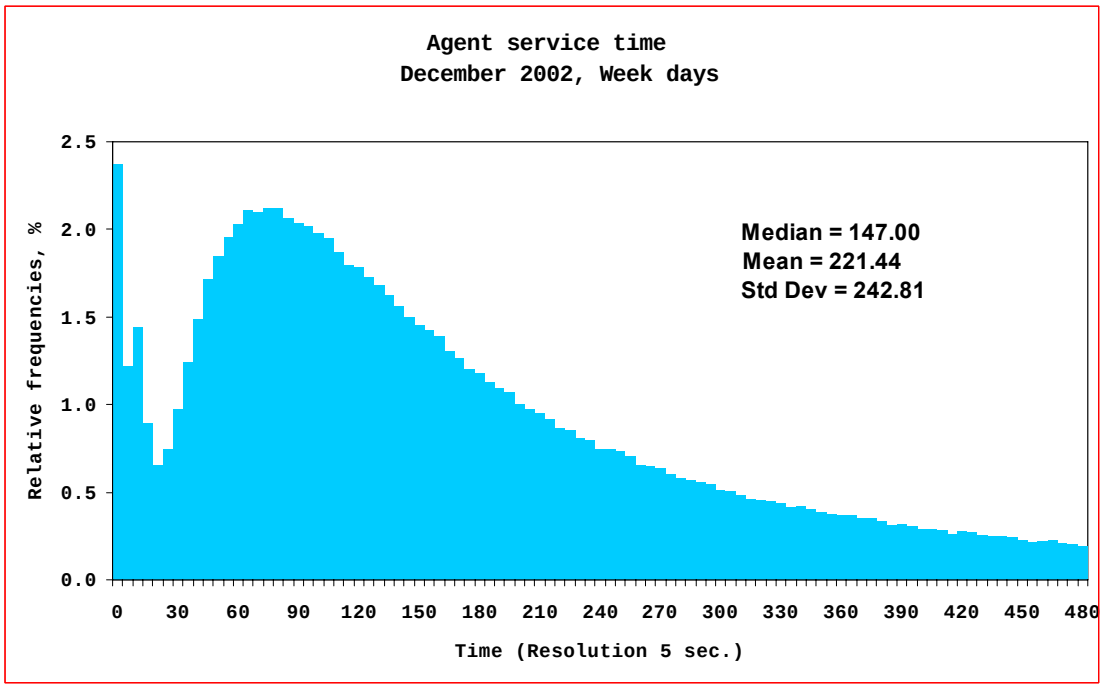


Figure 33 presents the distribution of agent service time for EBO calls in December 2002. Note unusual distribution with two modes.

Figure 33 : Distribution of agent service time for EBO calls (Dec-02)

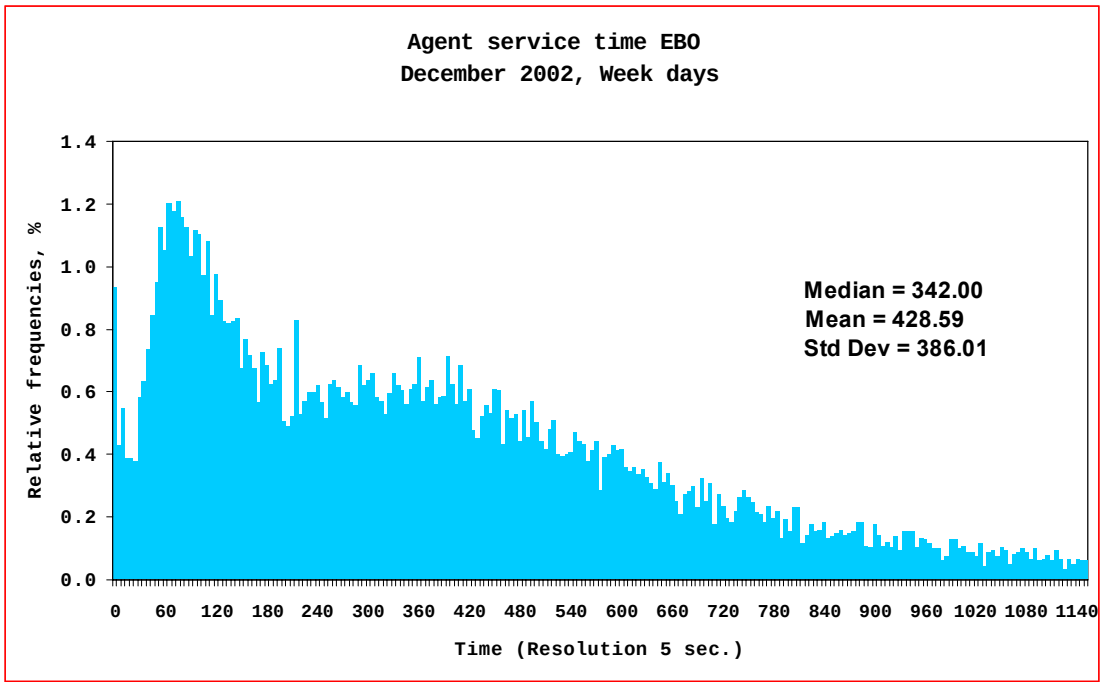
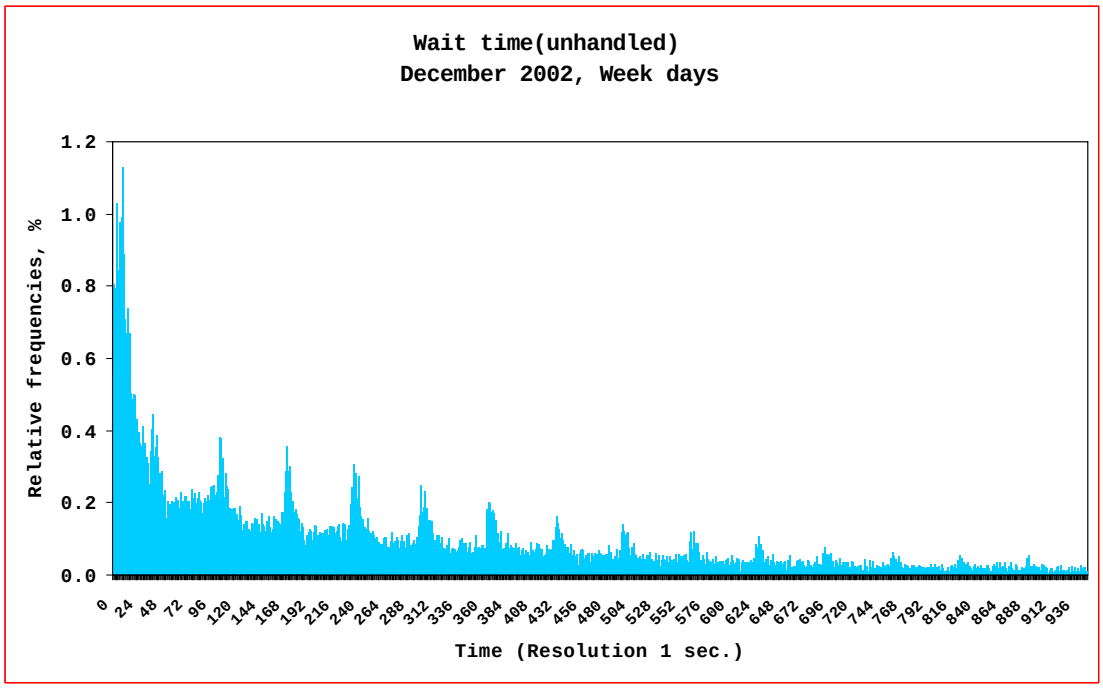


Figure 34 shows the distribution of Brokerage waiting times for callers who have to wait at all. Note that there are some (decreasing in size) modes at 31 sec (largest), 162

sec, 303 sec and 423 sec. The most likely correspond to responses to announcements or messages delivered while waiting.

Figure 34 : Distribution of unhandled for Brokerage calls (Dec-02)



4 VRU: starting the service process

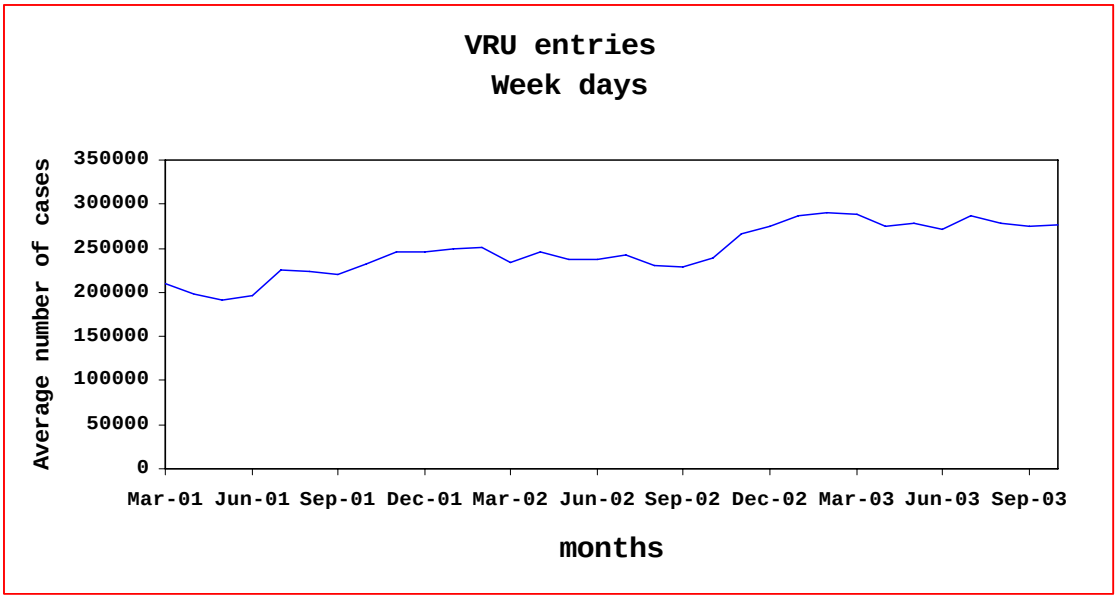
Most inbound calls must pass through the Voice Response Unit, or VRU. We can divide customers that enter the VRU into two types: those who do not ask for agents and those who, after completing the VRU interaction, enter the agents queue.

4.1 Arrivals to VRU

In this subsection we analyze the arrival process to the VRU.

Figure 35 shows the average number of customers that enter the VRU during weekdays. The whole period between March 2001 and October 2003 is studied. Note that the average number of VRU entries gradually increases during the considered period. The maximum is reached during March 2003.

Figure 35 : Average number of VRU entries for Total calls



Now consider the proportion of VRU-only entries for several types of calls. Note that not all types of customers can complete their service in the VRU. The following are these customer types: Platinum, Online Banking, EBO, Telesales, Case Quality, Priority Service, BPS.

Figure 36 shows the time series for the proportion of customers that receive service via the VRU and do not try to reach an agent (blue line). Note that this fraction is large (between 82% and 88%) and that a larger VRU-only proportion usually means saving costs since VRU service is typically much cheaper then service by agents. We see a gradual, overall, increase between March 2001 and December 2001.

Red line presents the fraction of customers that receive service via VRU and do not ask for agent service for Retail calls. As could be expected, this figure is similar to one for Total calls.

Yellow line shows the VRU-only rate for Consumer Loans calls. Note that the fraction of customers who do not ask for agents is smaller than for Retail. Green line displays VRU-only rate for Business calls. Here the percentage of customers who do not ask for agents is much larger than the overall average and it remains quite stable over the considered period.

Figure 36 : Proportion of VRU-only entries

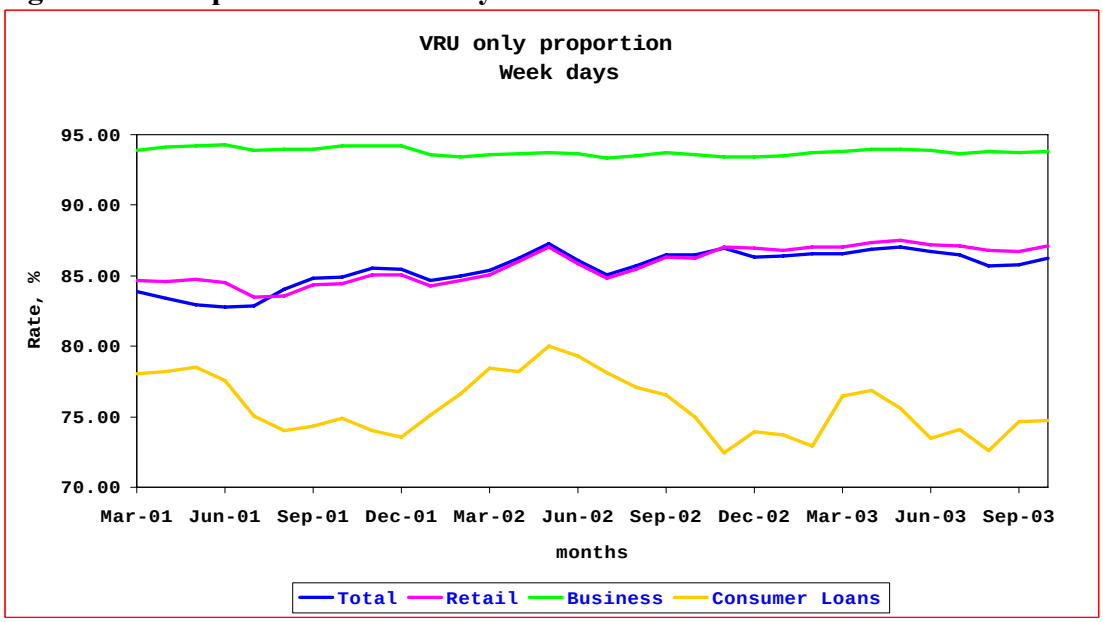
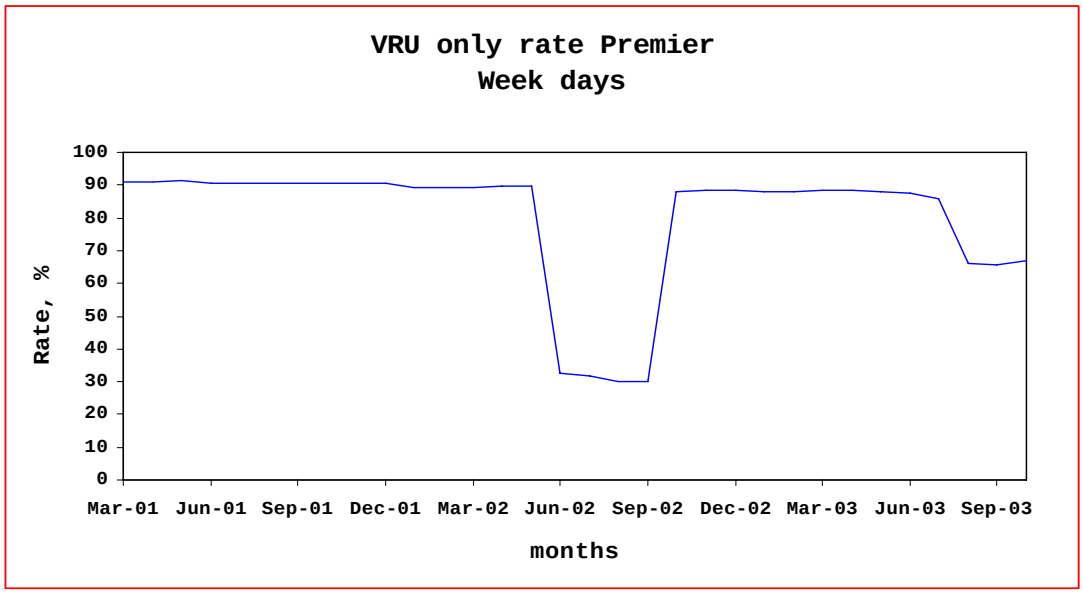


Figure 37 presents the VRU-only rate for Premier calls, note a sharp unexplained decrease between June 2002 and September 2002.

Figure 37 : Proportion of VRU-only entries for Premier Calls



Studying Premier calls in detail, we observe an interesting phenomenon. Figure 38 shows that VRU entries in the period from June 2002 till September 2002 fall almost to zero. Meanwhile, the average number of arrivals to agents for Premier calls does not decrease during these months; see Figure 62 in Subsection 5.2. The explanation is

a technical one and is due to the fact that in these months Premier calls entered the system in a non-standard way (not via VRU) and so were not counted within the VRU calls.

Figure 38 : Average number of VRU entries for Premier calls

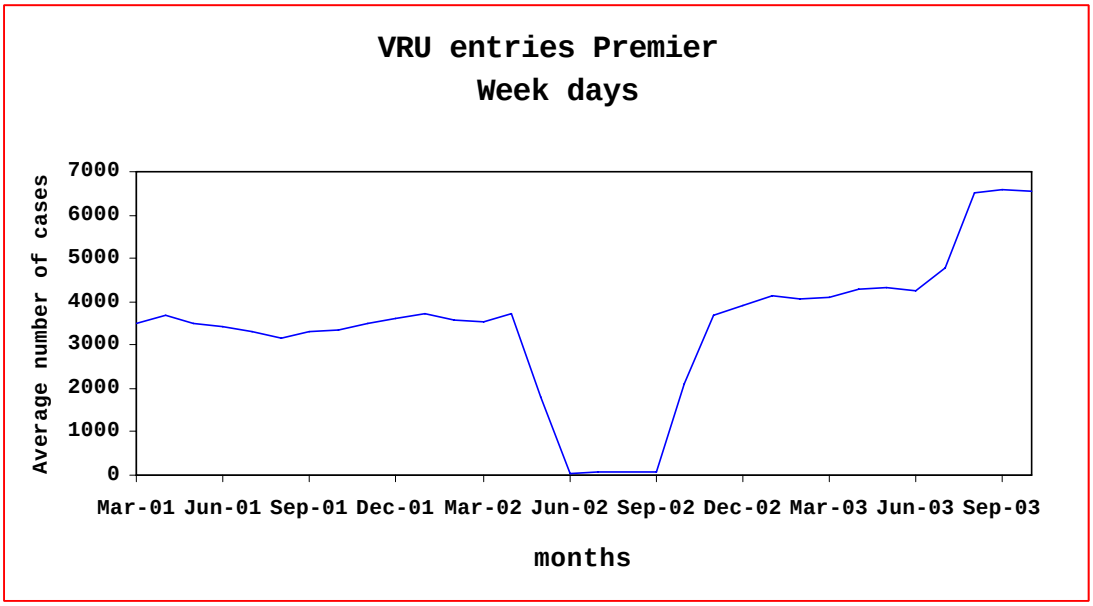
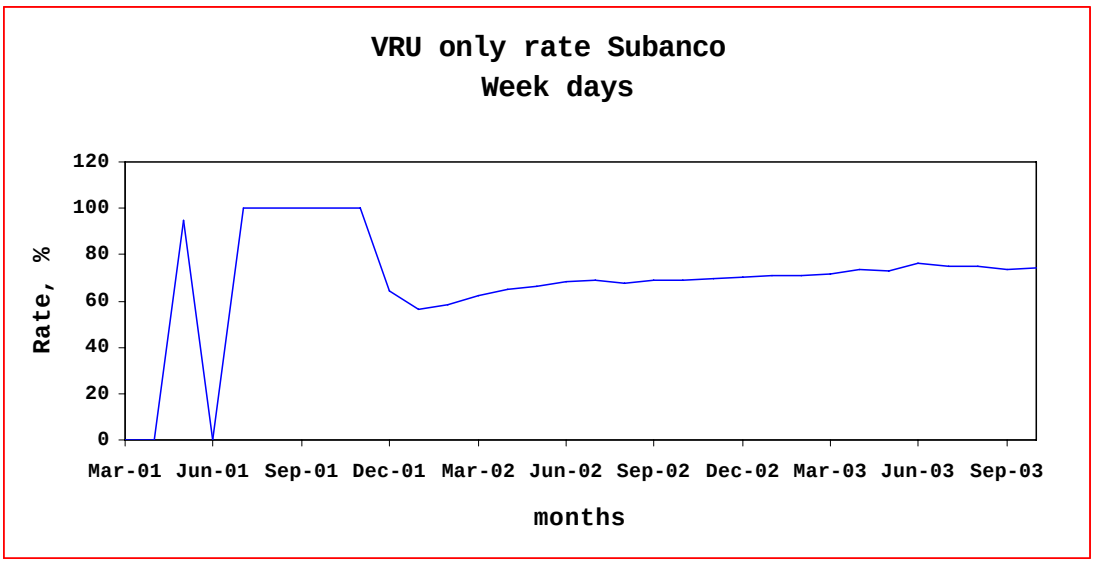


Figure 39 describes VRU-only rate for Spanish calls. We see another unusual phenomenon. Note that the VRU-only rate is 100% for period July – November 2001. This would mean that no customers of this type enter the agents queue. Meanwhile, Figure 63 from Subsection 5.2 contradicts this conclusion, indicating that there is some error in the coding of Spanish calls arriving to the VRU during this period.

Figure 39 : Proportion of VRU-only entries for Spanish Calls



4.2 Distribution of VRU-times

In this subsection we consider the distribution of time that customers spend in the VRU. *We consider only those customers who, after the VRU, attempted to reach an agent.*

Figure 40 describes the distribution of VRU-times for Total calls. We see that the distribution has two large peaks around 7 and 22 seconds and a small peak around 110 seconds.

Figure 40 : VRU-time for Total calls (May-01)

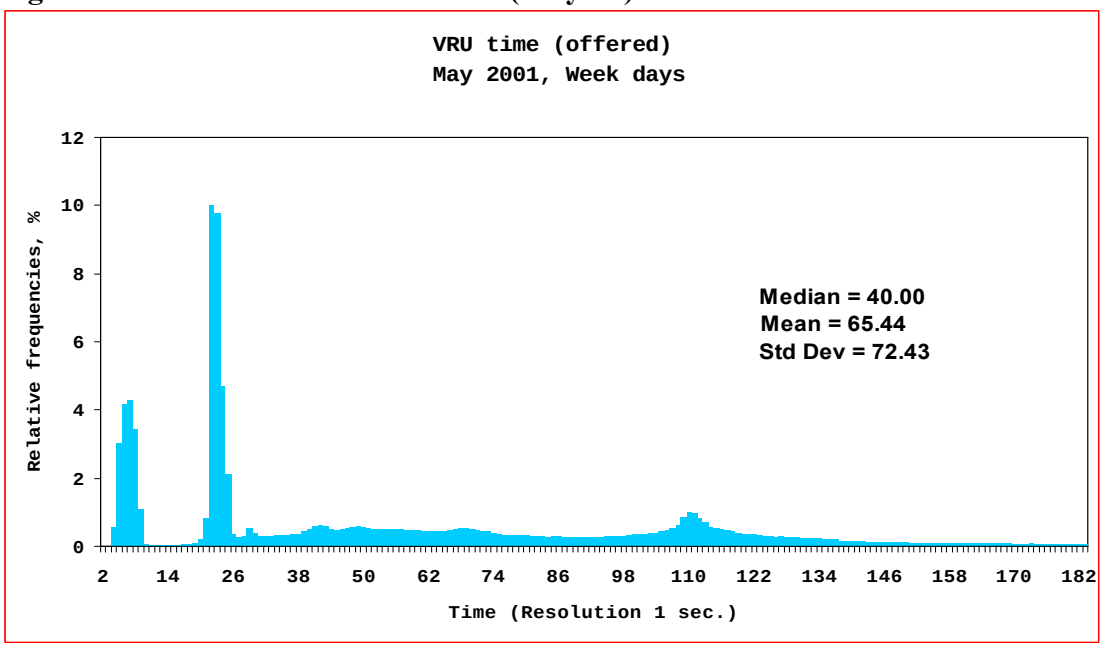


Figure 41 displays the distribution of VRU times for Retail calls. We observe again the last two peaks as shown in Figure 40, but the first peak does not show up.

Figure 41 : VRU-time for Retail calls (May-01)

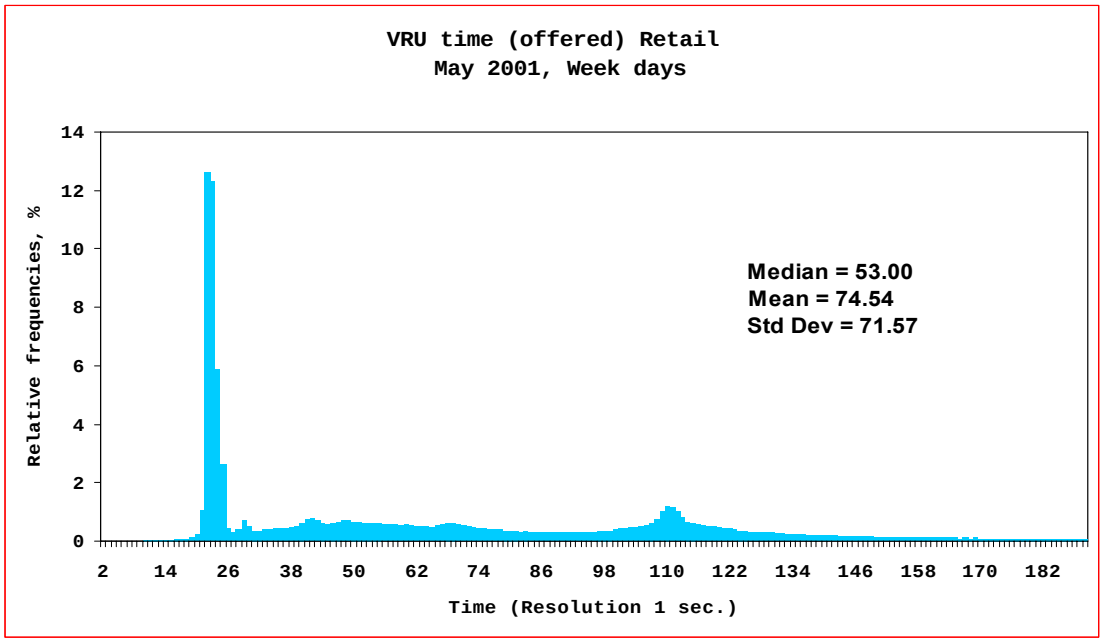
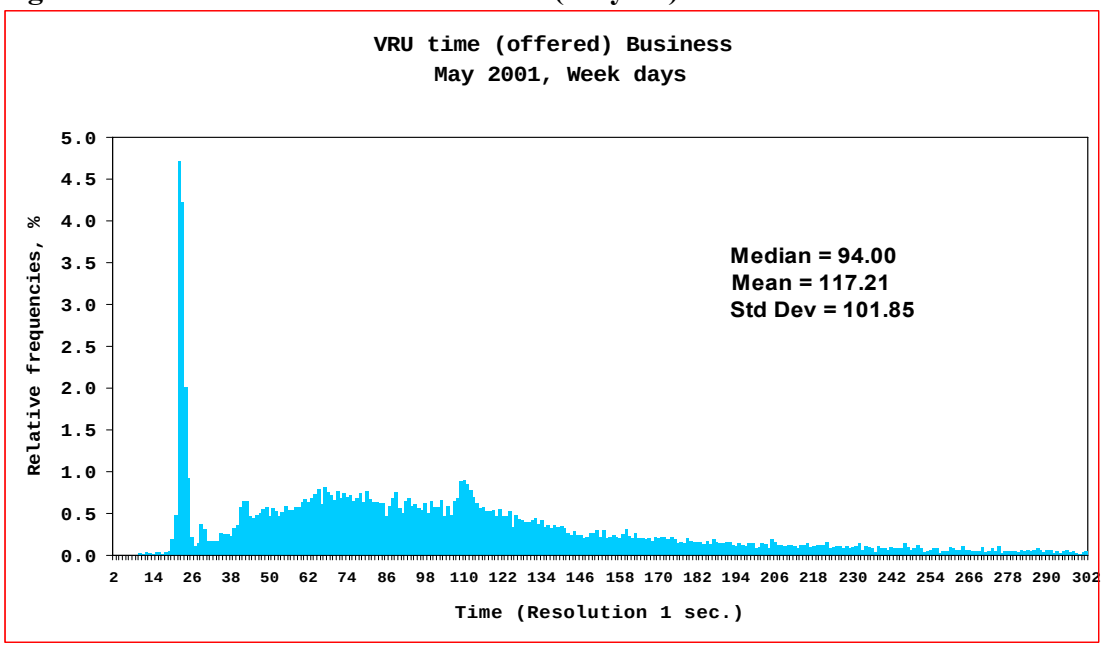


Figure 42 describes the distribution of VRU times for Business calls. A single significant peak around 22 second is observed.

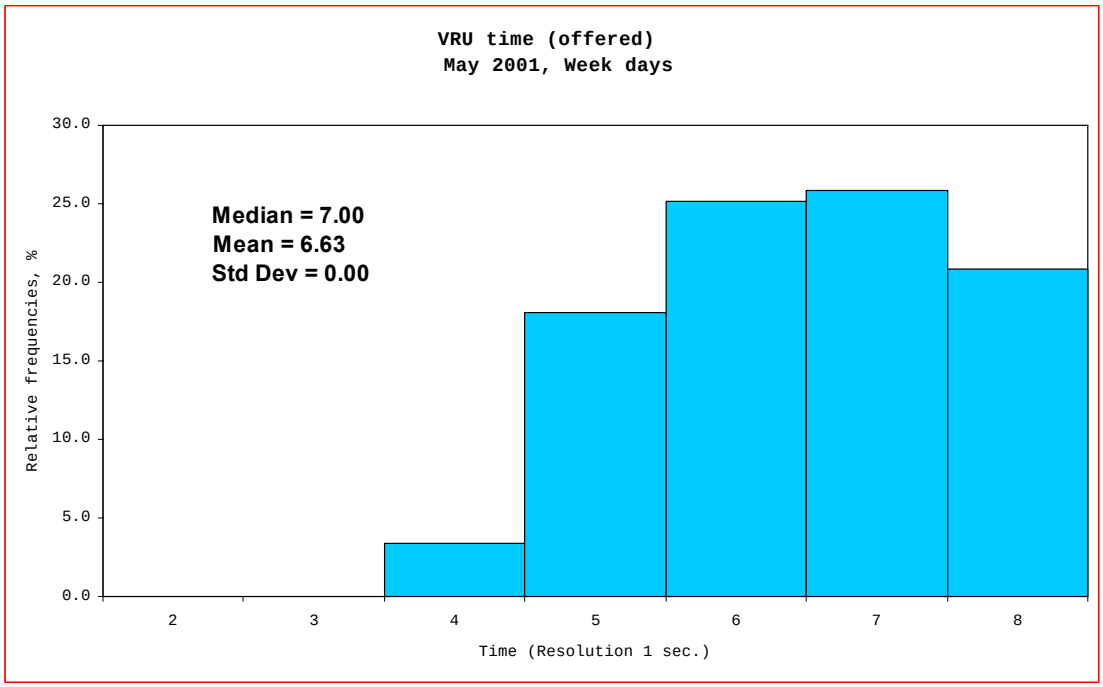
Figure 42 : VRU-time for Business calls (May-01)



Note that the location of the peaks for Retail and Business customers is identical. The meaning of the 22-second peak could be the length of an announcement at the beginning of the VRU service.

The first peak of the VRU time's distribution (at 7 seconds), as shown in Figure 40, is due to the Acquired Bank customers (see Figure 43).

Figure 43 : VRU-time for Acquired Bank calls (May-01)



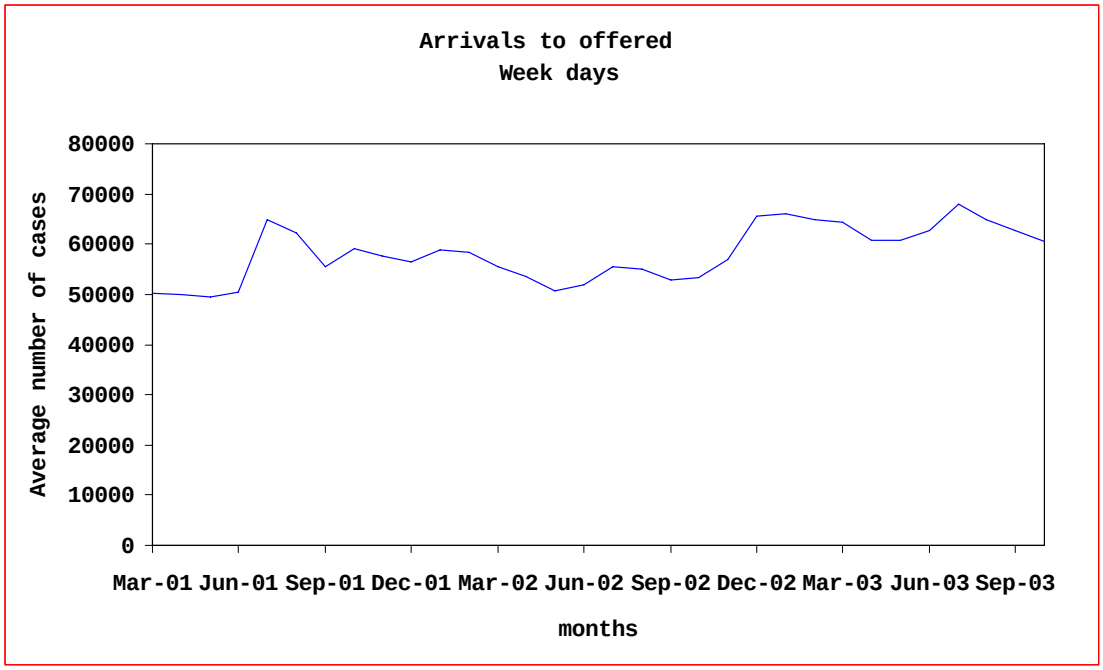
5 The arrival process – offered volume

5.1 Hierarchical profiles

Here we study the arrival process of customers that wish to talk to an agent. We call it the *offered volume* of calls requesting agent service. This process will now be described using four levels of representation, which differ by their time-scale. Figure 44 is a top-level yearly picture with month as the time unit. Figure 45 - Figure 49 are monthly plots with day as the time unit. Figure 50 - Figure 54 are daily pictures with hours as the time unit. Finally, Figure 55 - Figure 57 are hourly pictures with minutes as the time unit.

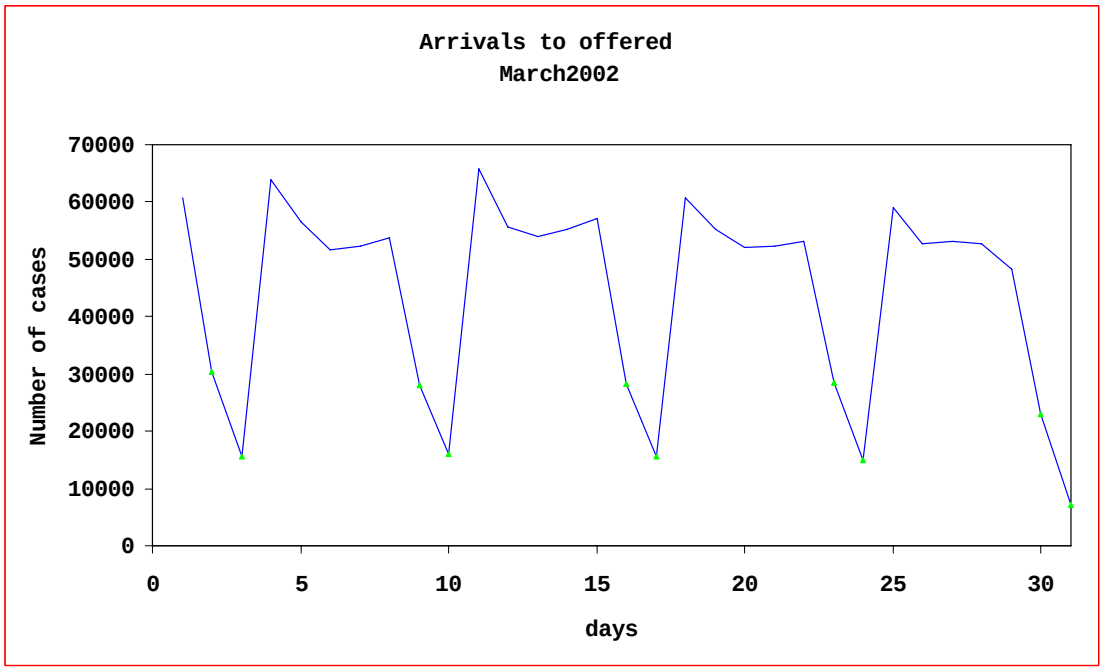
Figure 44 shows the average number of calls per weekday for the period from March 2001 to October 2003. Towards the end of the period, we observe a moderate increase in the number of calls.

Figure 44 : Average number of calls per weekday



The next level displays the overall number of calls per day over a month. March, 2002 is considered in Figure 45. The "valleys" occur during weekends.

Figure 45 : Number of calls per day (Mar-02)



The patterns of other months are similar (see Figure 47, Figure 48 and Figure 49), with additional valleys during holidays. For example, Figure 46 describes arrivals in December 2001. Note the very low arrival rate on Christmas, December 25.

Figure 46 : Number of calls per day (Dec-02)

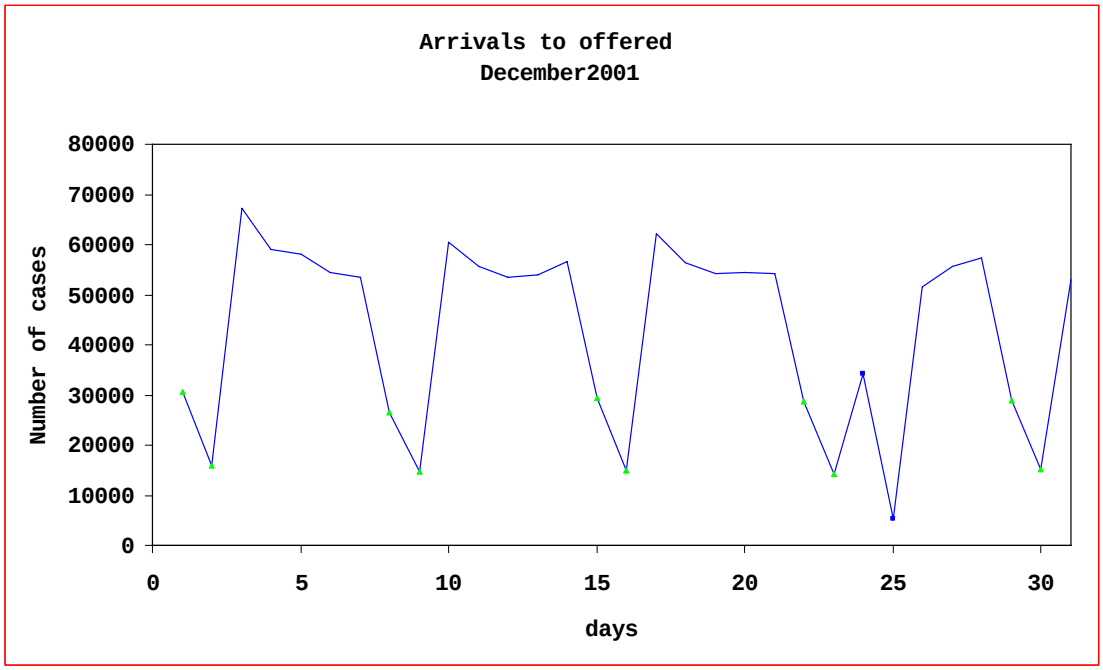


Figure 47 : Number of calls per day (2001)

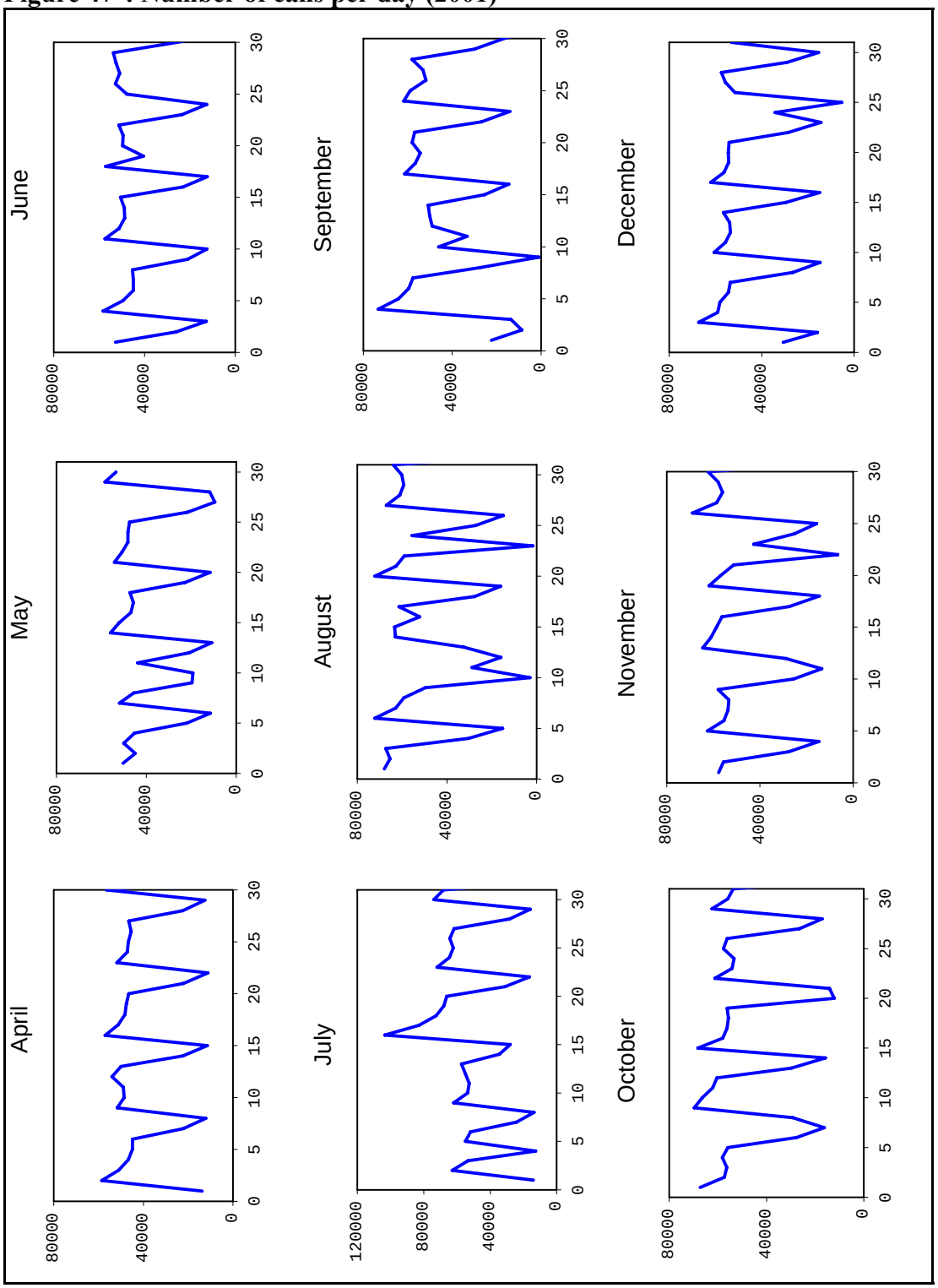


Figure 48 : Number of calls per day (2002)

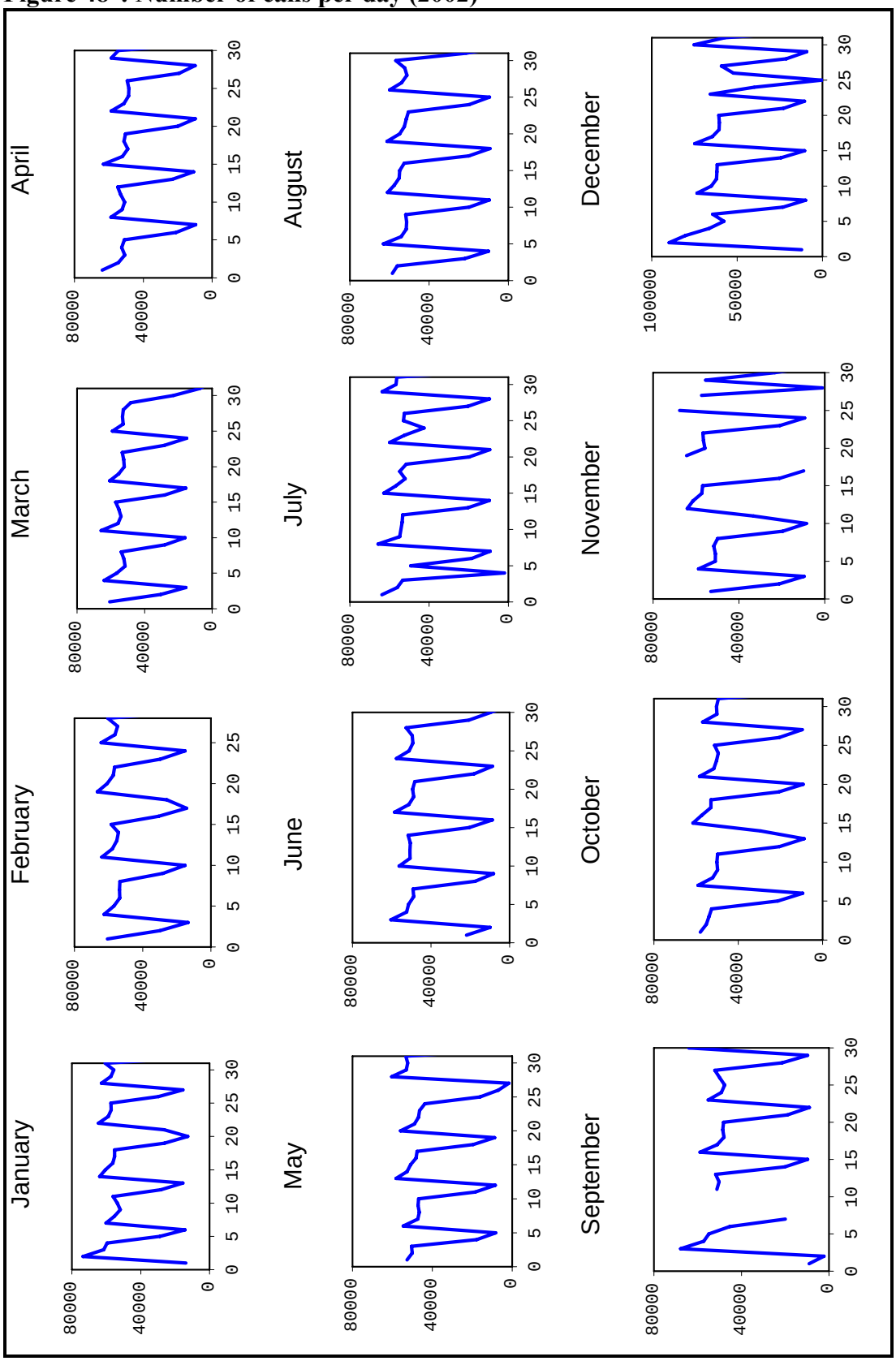
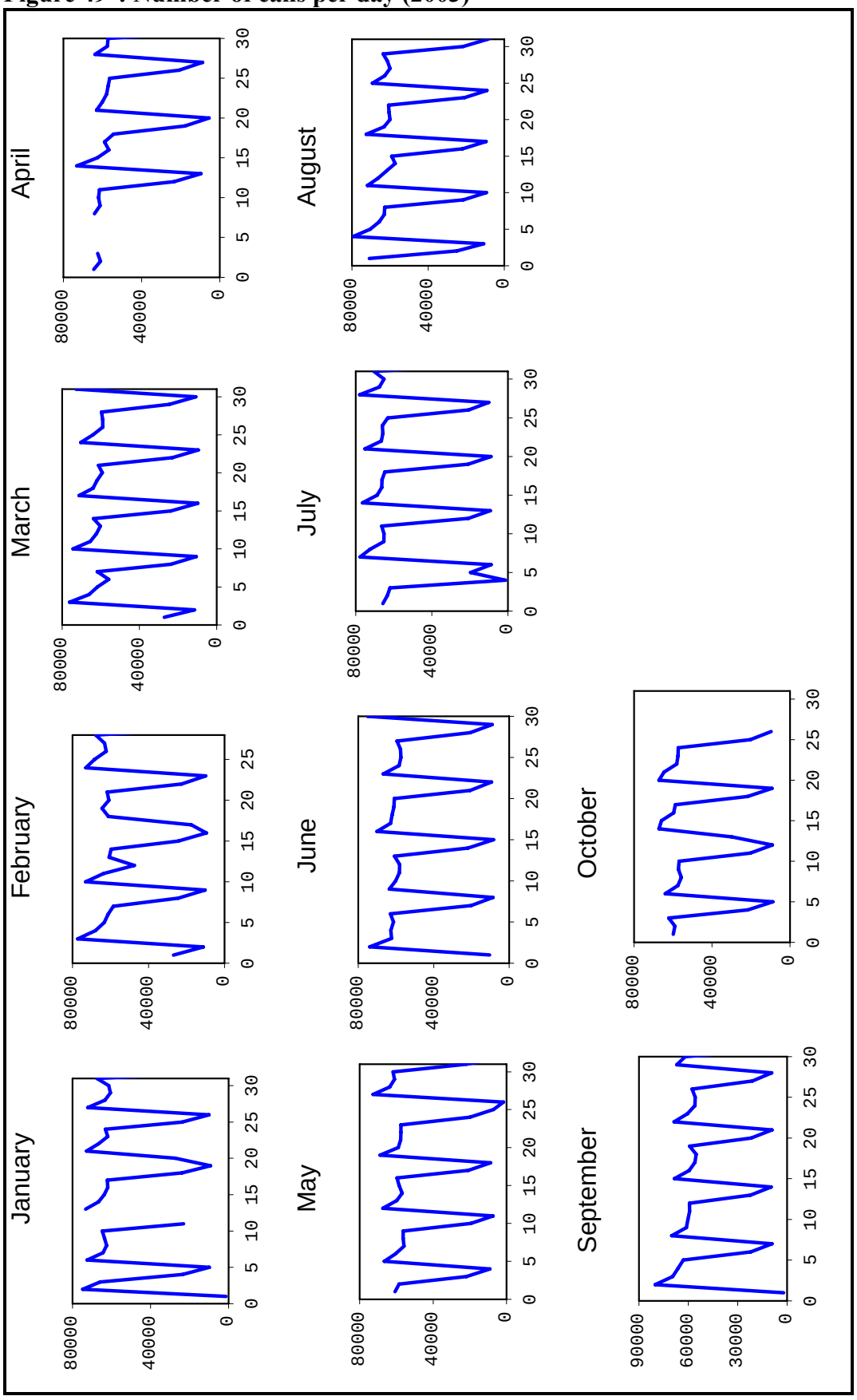


Figure 49 : Number of calls per day (2003)



The next level displays the average number of calls per hour. Figure 50 shows an hourly call volume during weekdays in March, 2002. The peak of the arrival rate takes place around 9-10 am, then, after a slight decrease, the number of arriving calls stabilizes. It starts to decrease steadily again around 4pm.

Figure 50 : Average number of calls per hour (Mar-02)

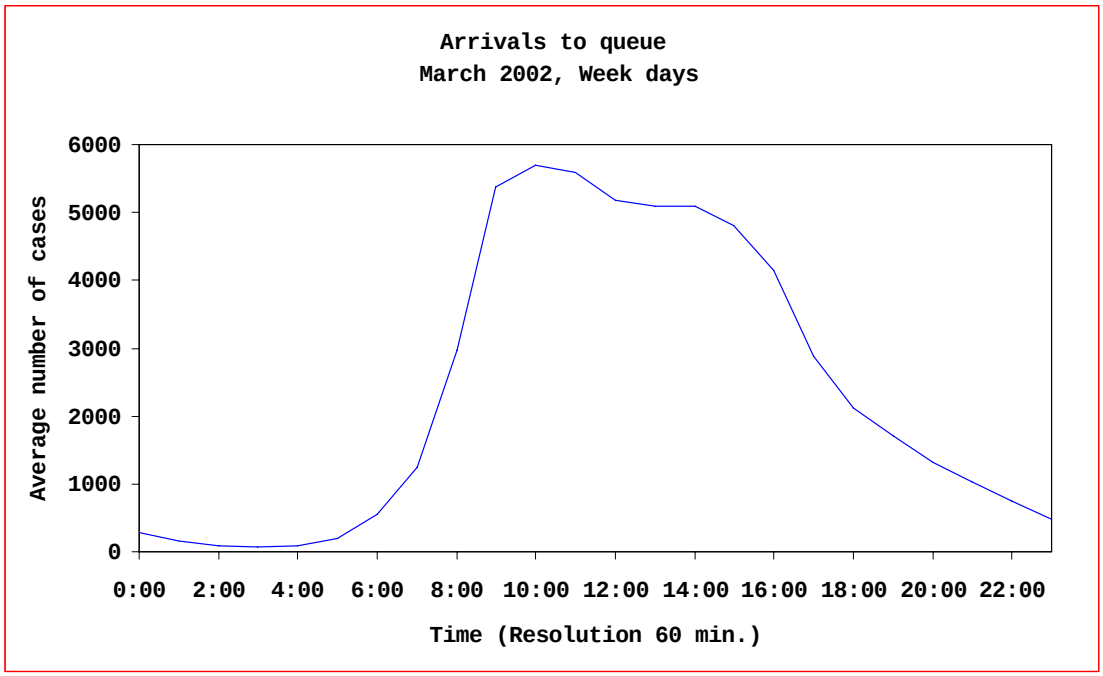


Figure 51 shows that the average number of calls on Mondays is, in general, larger than the average number of calls on other weekdays.

Figure 51 : Average number of calls per hour (Mar-02)

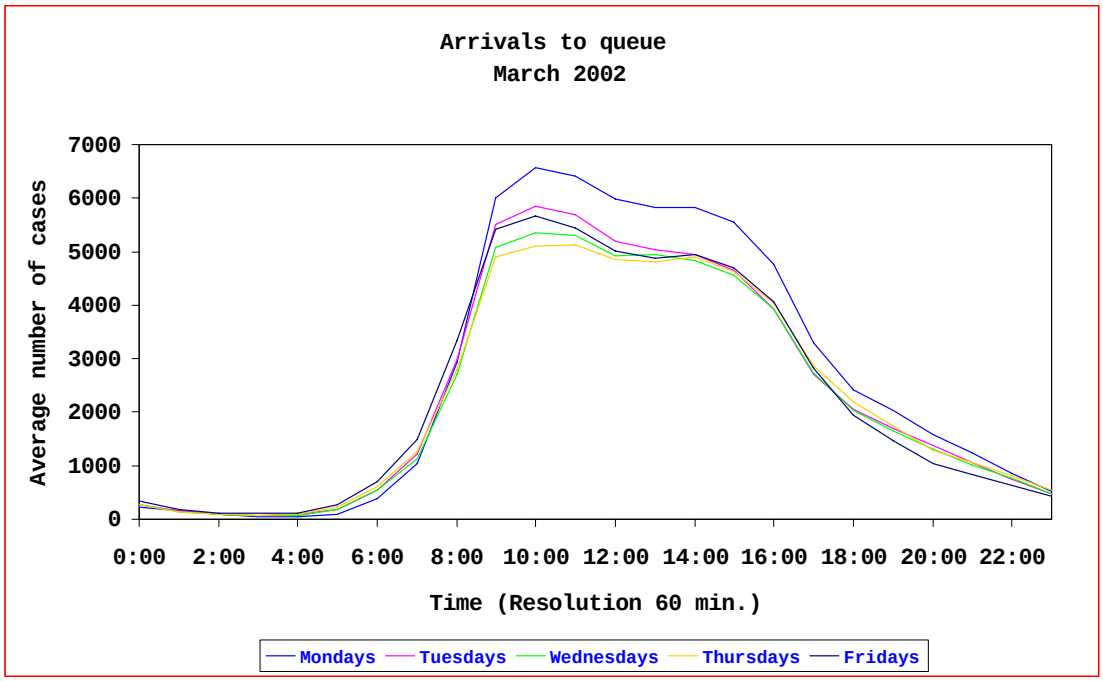


Figure 52 describes the dynamics of the average number of calls per hour for May 2001, May 2002 and May 2003. Note that the three patterns are similar and the curves almost coincide for May 2001 and May 2002.

Figure 52 : Average number of calls per hour (May-01, May-02 and May-03)

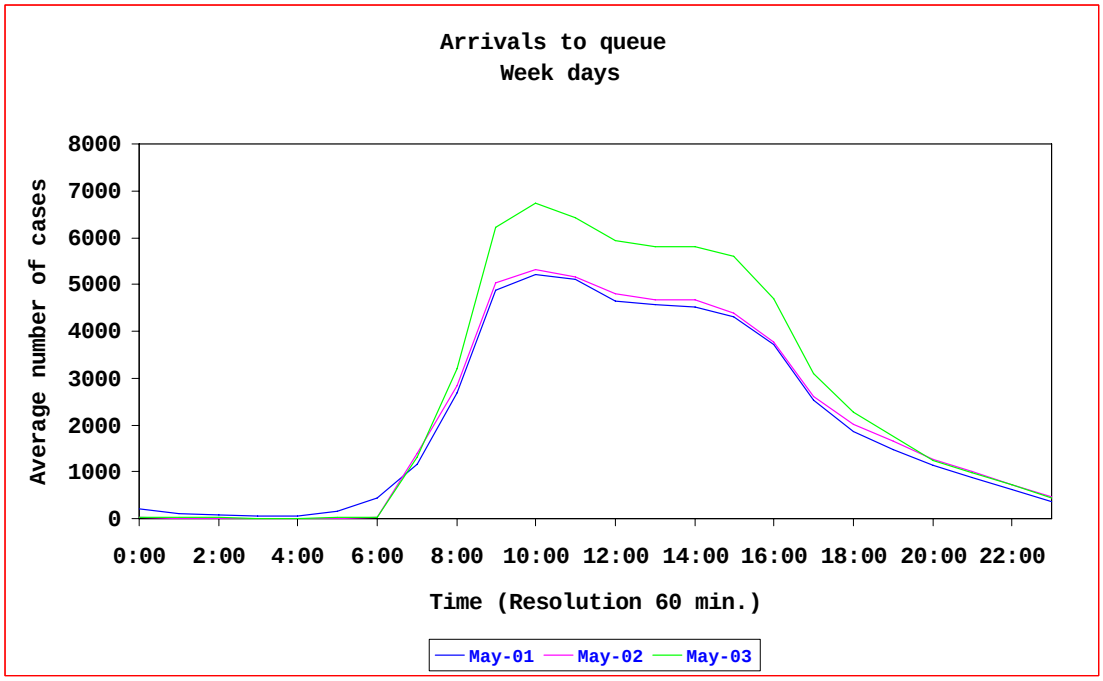


Figure 53 presents percent to mean of number of calls in March 2002. Note that the pattern is similar to Figure 51.

Figure 53 : Number of calls per hour, percent to mean (Mar-02)

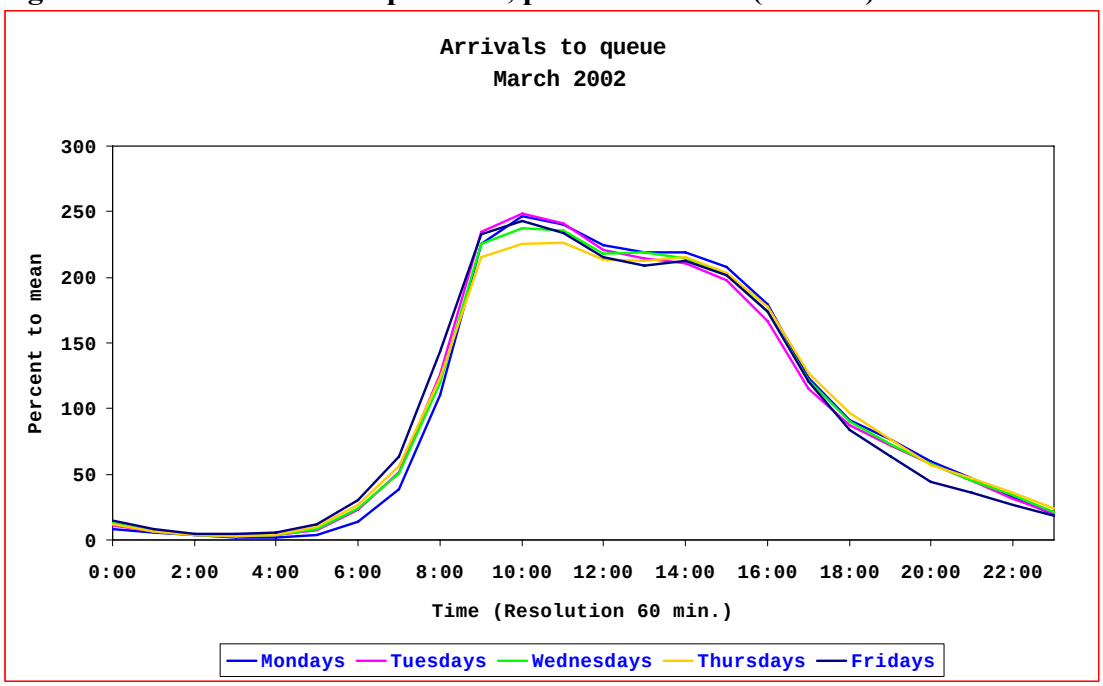
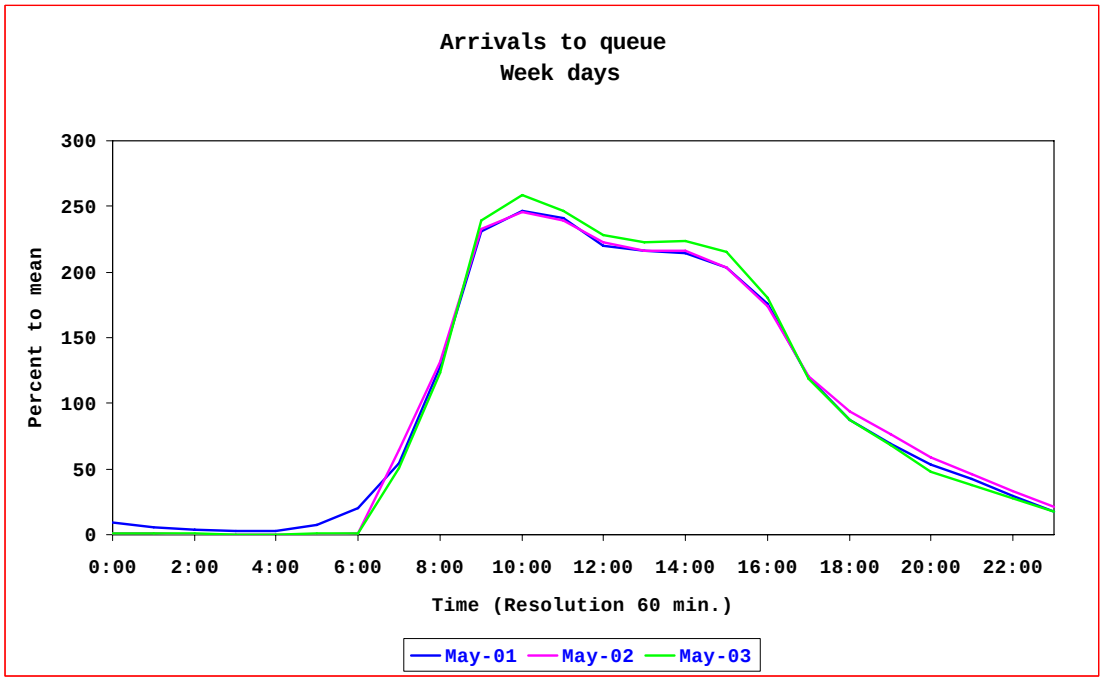


Figure 54 shows percent to mean of number of calls per hour for May 2001, May 2002 and May 2003. We observe that the pattern is similar to Figure 52.

Figure 54 : Number of calls per hour, percent to mean (May-01, May-02 and May-03)



Finally, if one looks at an individual hour, calls seem to arrive randomly. Figure 55 displays the number of calls per minute that arrive between 10am and 11am during a typical day in March 2002 (March 12th). The Poisson process with a constant arrival rate could be an appropriate model for this hour ([2]).

Figure 55 : Number of calls per minute (Tuesday, March 12, 2002)

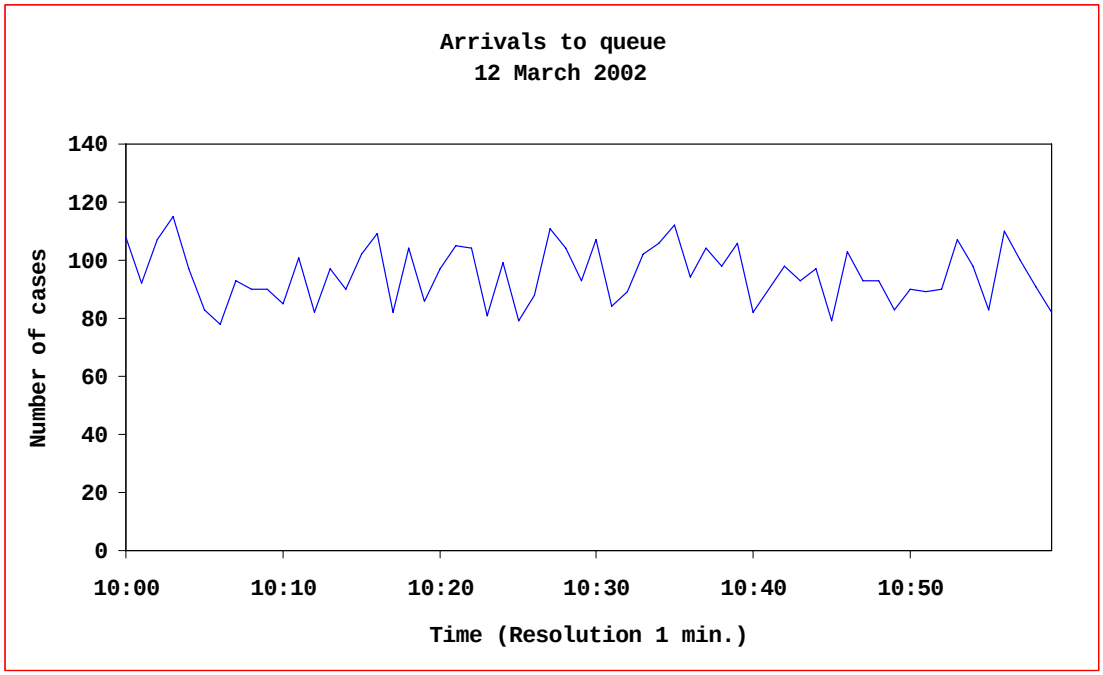


Figure 56 and Figure 57 present call volumes per hour and per minute, respectively, during weekdays in March 2002 with resolution 30 seconds.

Figure 56 : Average number of calls per hour (resolution 30 seconds)

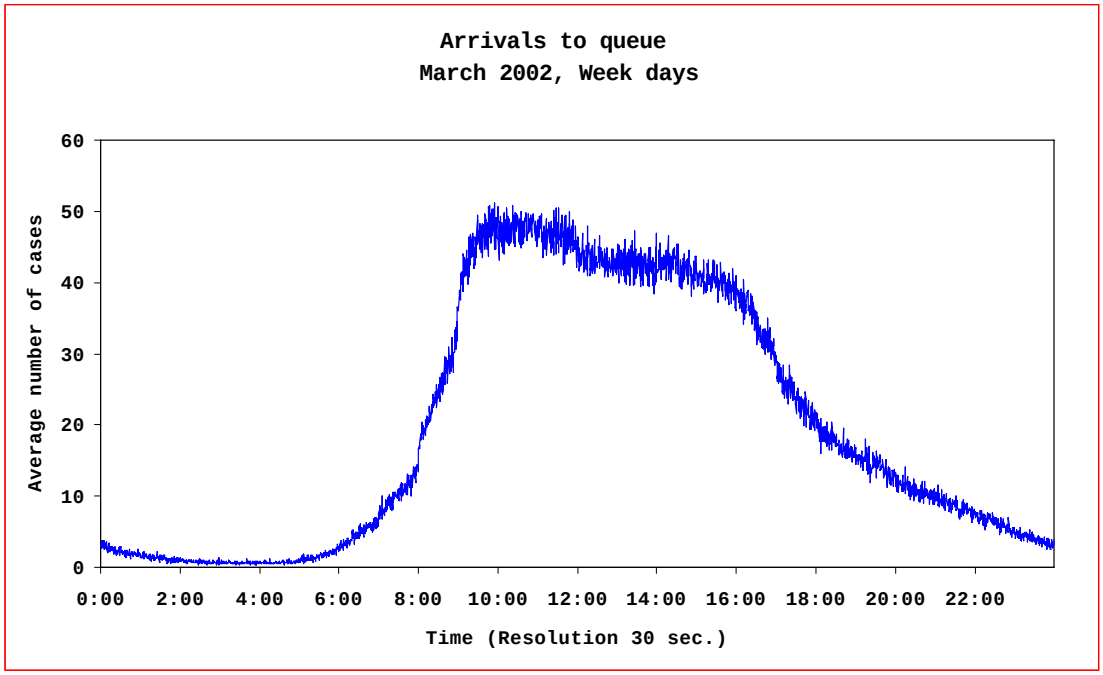
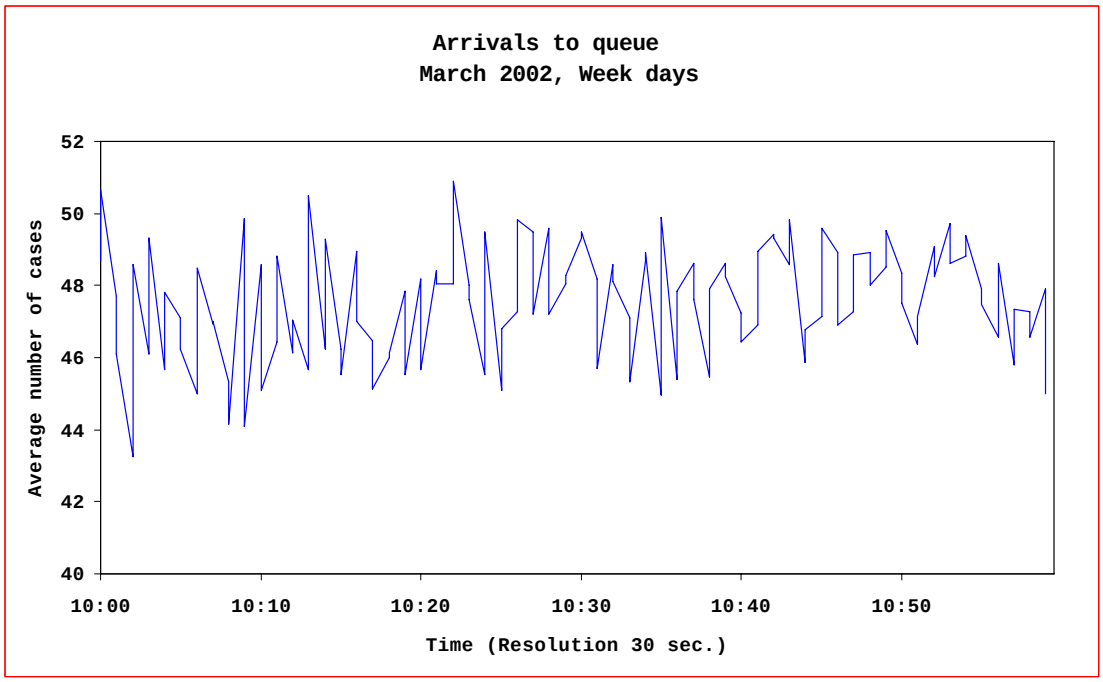


Figure 57 : Average number of calls per minute (resolution 30 seconds)



5.2 Customer profile counts

It is of interest to analyze the distribution of calls according to different customer types. There are 17 types of customers:

- Retail - the most prevalent type in this call center
- Premier – high priority customer of Retail type
- Business

- Platinum - high priority customer of Business type
- Consumer Loans
- Online Banking
- EBO – Emergency Banking Operation (for example, loss of credit card)
- Telesales
- Spanish – service in Spanish language
- Acquired Bank – exists as independent unit until July 2001
- Case Quality – exists from July 2002
- Priority Service - exists from May 2002
- Brokerage – brokerage, was integrated with the main system from November 2002
- BPS – business solutions, exist from November 2002
- AST – Advance Solution Teams, exist from November 2002
- CCO – Credit Card Operations exist until February 2002 and appears again from November 2002
- Mortgage – exist in the period from November 2002 until January 2003

Table 10, Table 11, and Table 12 summarize the arrival process for each type. They display average number of customers of different types during 2001, 2002 and 2003.

Table 10: Daily arrival rate by customer type on weekday (2001)

Type	Mar-01	Apr-01	May-01	Jun-01	Jul-01	Aug-01	Sep-01	Oct-01	Nov-01	Dec-01
Total	50305.0	49892.4	49505.6	50405.9	64922.5	62280.1	55542.2	59204.5	57683.4	56394.2
Retail	31859.2	30973.4	30231.6	31561.0	45325.1	45043.2	40161.8	41276.6	40419.6	41075.1
Premier	1912.2	1967.3	1838.9	1835.8	1903.4	1832.2	1850.3	1775.6	1717.8	1827.4
Business	3556.6	3398.5	3200.8	3022.6	3483.2	3565.1	3574.4	3608.4	3581.1	3593.5
Platinum	272.6	268.4	277.0	268.1	288.6	298.0	325.5	320.6	319.0	322.0
Consumer Loans	2914.8	2984.1	2888.1	2748.9	3548.2	4317.0	3741.8	4497.5	4232.1	3785.5
Online Banking	1426.2	1337.4	1253.1	1401.8	2234.1	2059.1	1790.8	1918.5	1864.7	1870.2
EBO	554.8	508.1	458.5	414.3	512.6	528.7	524.8	518.8	509.3	520.6
Telesales	4504.0	4111.3	3812.1	3670.1	4287.1	4352.8	3338.1	4764.5	3670.2	2512.9
Spanish	125.0	122.6	142.0	140.0	269.6	283.9	234.8	267.3	273.6	200.8
CCO	0.0	0.0	0.0	0.0	0.0	0.0	0.0	256.7	1095.9	686.2
Acquired Bank	3179.6	4221.4	5403.5	5343.3	3070.5	0.0	0.0	0.0	0.0	0.0

Table 11 : Daily arrival rate by customer type on weekdays (2002)

Type	Jan 02	Feb 02	Mar 02	Apr 02	May 02	Jun 02	Jul 02	Aug 02	Sep 02	Oct 02	Nov 02	Dec 02
Total	58901.0	58282.8	55510.4	53533.2	50632.7	51928.5	55443.8	55119.5	52873.3	53346.3	57035.6	65653.0
Retail	43024.7	42717.3	40612.8	39001.8	36756.8	37829.7	39836.2	39486.0	37036.3	36487.9	35105.8	35592.1
Premier	1952.6	1848.7	1846.4	1851.5	1669.9	1790.6	1891.4	1781.6	1776.8	1783.0	1891.8	1933.8
Business	3816.4	3682.8	3501.2	3376.6	3421.1	3449.3	3431.4	3331.6	3238.6	3266.9	3224.0	3372.8
Platinum	351.6	335.4	302.9	297.5	293.6	291.9	283.1	273.8	269.6	270.4	259.5	260.1
Consumer Loans	3935.4	3774.4	3545.1	3336.0	3098.4	3186.9	3383.3	3467.0	3639.9	3768.7	4037.8	3589.9
Online Banking	2050.7	1894.4	1827.5	1701.0	1555.3	1619.8	1764.4	1712.7	1881.9	1723.9	1734.3	1752.4
EBO	517.0	487.9	448.3	424.5	426.5	419.8	437.9	446.2	462.7	467.7	479.3	503.0
Telesales	2674.4	3310.5	3196.7	3322.3	3167.4	3084.9	3872.6	3773.6	3505.2	3967.9	3472.6	2744.4
Spanish	199.7	222.1	229.6	222.0	225.2	226.7	275.4	316.5	312.0	321.0	323.6	317.9
Case Quality	0.0	0.0	0.0	0.0	0.0	0.0	69.2	183.9	360.7	581.4	677.0	622.3
Priority Service	0.0	0.0	0.0	0.0	18.5	28.9	199.0	346.5	389.7	707.5	798.5	727.3
AST	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	266.1	1519.1
CCO	378.6	9.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	738.0	3790.4
Brokerage	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3393.2	8268.5
Mortgage	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	518.5	196.4
BPS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	115.6	462.6

Table 12 : Daily arrival rate by customer type on weekdays (2003)

Date	Jan 03	Feb 03	Mar 03	Apr 03	May 03	Jun 03	Jul 03	Aug 03	Sep 03	Oct 03
Total	65955.1	64895.1	64277.0	60785.0	60792.0	62671.7	68052.5	64900.5	62625.9	60546.8
Retail	36194.2	36240.1	35394.6	32904.3	32782.8	33358.2	35044.5	34631.7	34068.3	33327.2
Premier	2092.3	2045.2	2137.4	2090.7	2058.1	2115.8	2382.2	2261.2	2248.5	2129.5
Business	3374.7	3261.2	3220.0	2999.3	2993.0	2974.5	3103.1	2948.0	3148.2	3006.1
Platinum	266.8	246.4	241.9	224.1	222.0	217.2	215.0	221.0	234.3	222.8
Consumer Loan	4000.9	3933.5	4001.3	3851.3	3950.8	4399.8	4689.8	4259.8	3957.8	3556.2
Online Banking	1986.8	1808.9	1806.5	1702.7	1571.0	1581.4	1839.7	1969.3	1809.0	1699.9
EBO	499.2	490.9	446.0	443.5	466.0	450.6	453.8	434.3	460.6	481.6
Telesales	3410.9	3790.8	3479.2	3311.9	3451.5	3684.0	4717.4	4150.2	3370.1	3301.1
Spanish	339.5	363.4	358.8	330.9	345.2	299.7	320.8	333.3	354.1	360.8
Case Quality	618.3	651.7	709.8	658.8	610.8	631.4	736.4	730.1	780.9	781.0
Priority Service	882.8	1075.8	1166.5	1111.9	1057.2	1104.1	1358.9	1221.0	1228.6	1320.5
AST	2289.2	2194.9	2090.5	2087.3	2105.7	2179.4	2161.5	2170.3	2240.4	2174.2
CCO	3926.1	4230.4	5022.1	4828.5	5146.2	5301.5	6151.7	5601.0	4333.6	3872.6
Brokerage	5540.1	4108.9	3703.0	3755.9	3560.5	3861.0	4408.5	3502.0	3885.1	3763.1
Mortgage	84.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
BPS	448.9	453.1	499.4	484.0	471.2	513.1	469.4	467.3	506.5	550.2

Main observations:

- Retail calls constitute a very significant part of the total calls.
- The most prevalent types are Retail, Business, Consumer Loans, Telesales, Acquired Bank (until July 2001), Brokerage (from November 2002) and CCO (from December 2002).

- The low-volume types are Platinum, EBO, Spanish, Case Quality (from July 2002), Priority Service (from May 2002), Mortgage (During November 2002 – January 2003) and BPS (from November 2002).
- Significant increase of CCO (from 738 to 3790) and AST (from 266 to 1519) calls from December 2002 observed.

Figure 58 shows the dynamics of daily rates for Total and Retail calls. Note that Retail calls constitute a very significant part of the total calls. We also observe that the Retail calls percentage of the total arrival rate decreases after September 2002. Note also the decrease of Retail calls after July 2001. This trend can be explained by the increase in the fraction of customers who receive service via VRU and do not ask for agent service (see Figure 36).

Figure 58 : Average number of calls per weekday for Total and Retail calls

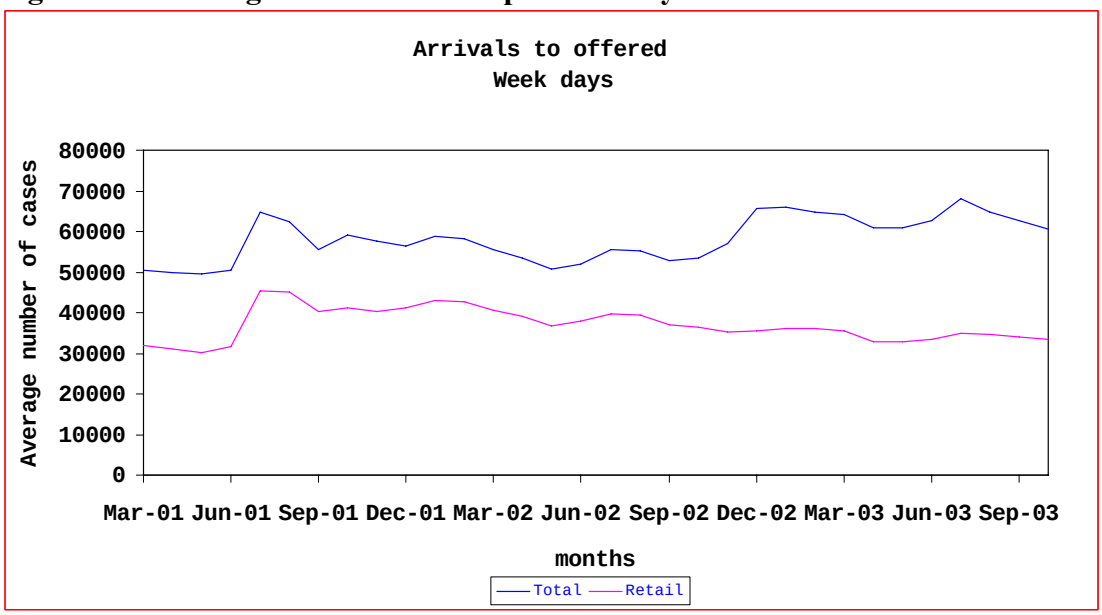
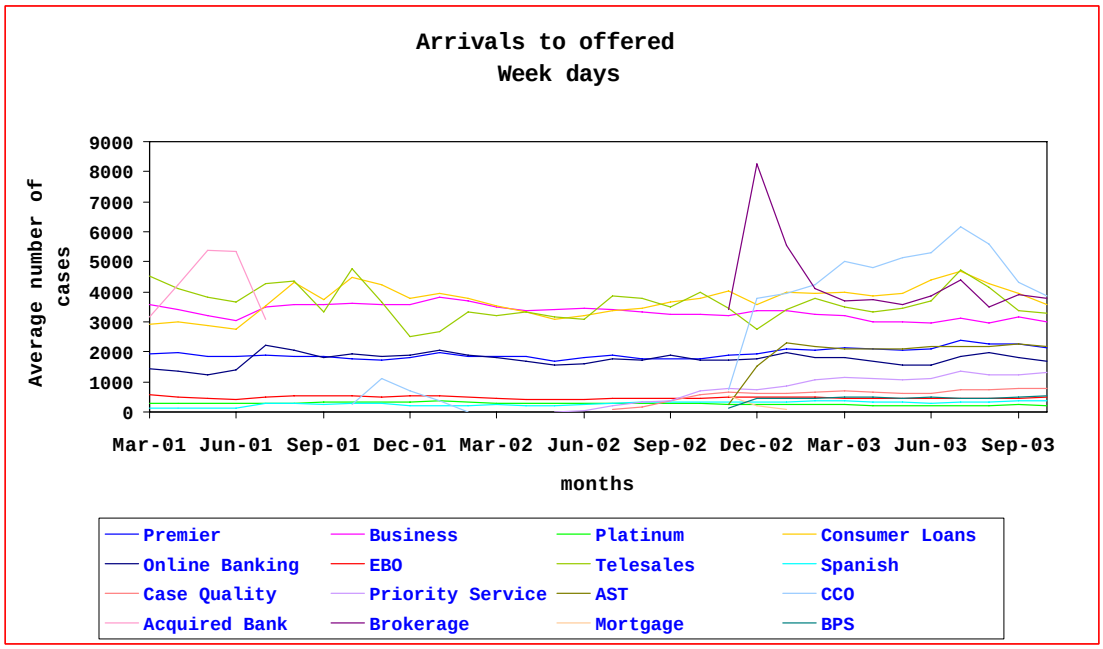


Figure 59 presents the monthly series of average number of calls per weekday for other customer types, apart from Retail calls.

Figure 59 : Average number of calls per weekday



Now we consider some customer types in detail. Figure 60 and Figure 61 show the monthly time series of weekday averages for Platinum and Business customer types. Note their decrease after January 2002.

Figure 60 : Average number of Platinum calls per weekday

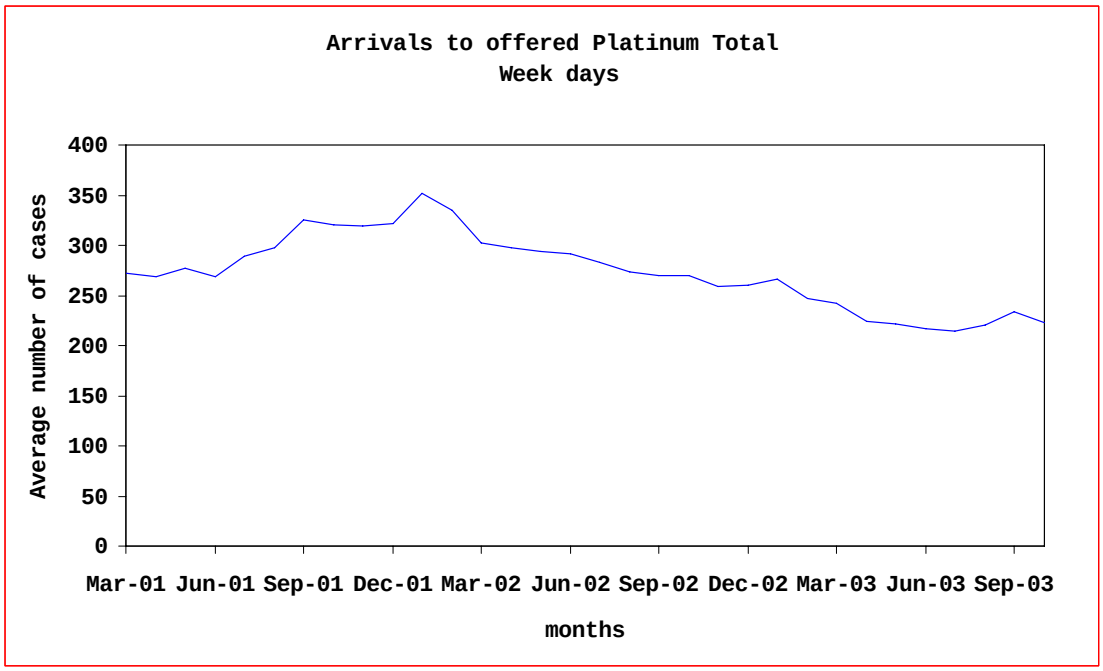


Figure 61 : Average number of Business calls per weekday

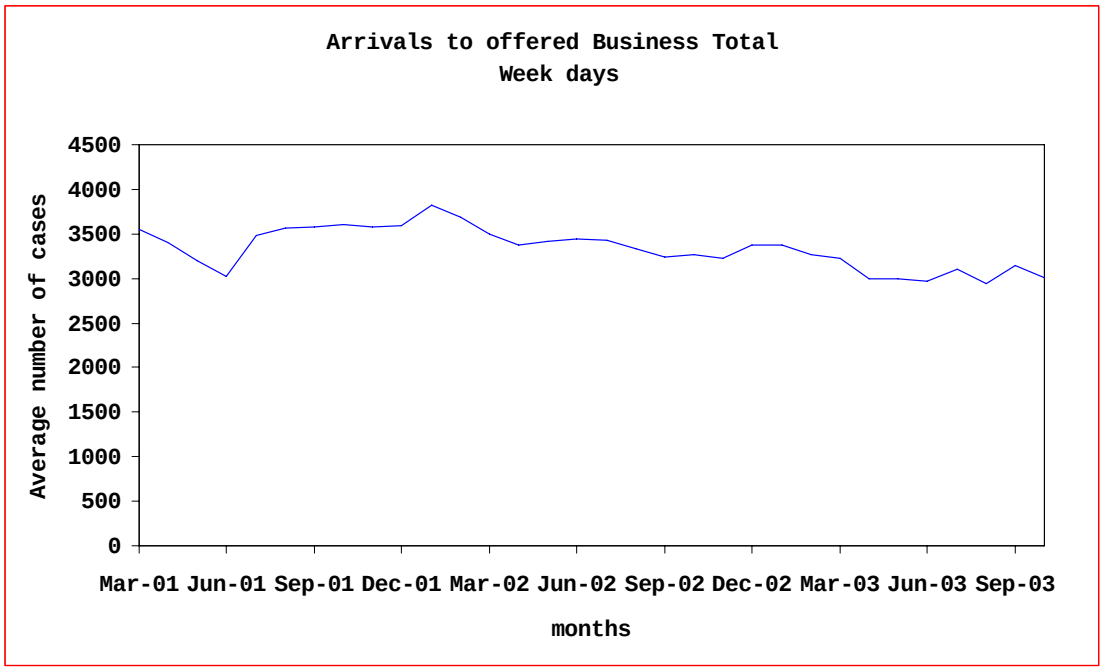


Figure 62 displays the average number of Premier calls per weekday. We observe that there appears to be an increasing trend towards the second half of the period.

Figure 62 : Average number of Premier calls per weekday

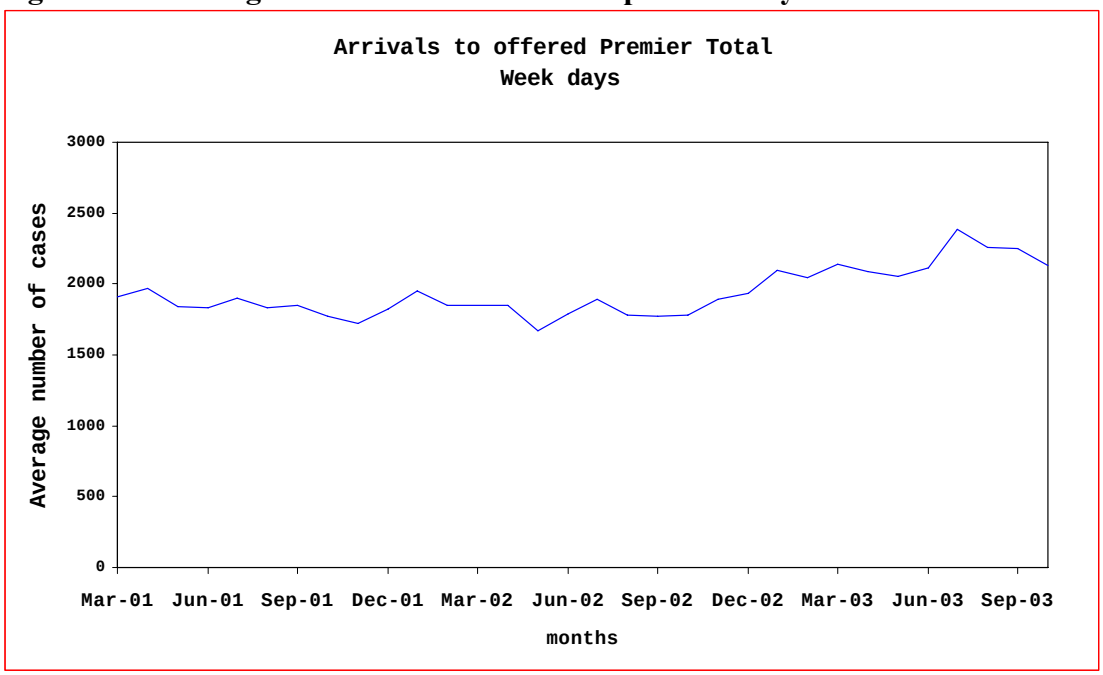


Figure 63 shows the average number of Spanish calls per weekday. We observe a decrease in the number of Spanish calls at December 2001, followed by a steady increase afterwards.

Figure 63 : Average number of Spanish calls per weekday

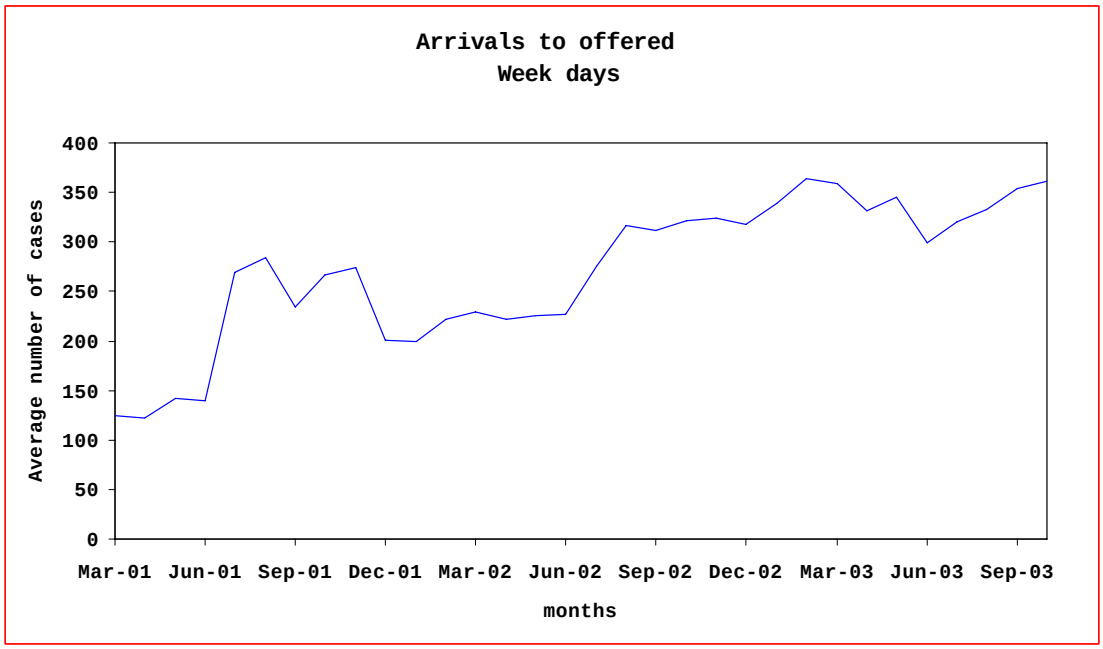


Figure 64 displays average number of Spanish calls in September 2001, September 2002 and September 2003. Note that arrival rate is zero after 5pm in September 2002 and September 2003, and it decreases significantly after 4:30pm in September 2001. Note also that patterns of arrival rates in the period from May 2001 till December 2001 are similar to September 2002 arrival rate pattern.

Figure 64 : Average number of Spanish calls (Sep-01, Sep-02 and Sep-03)

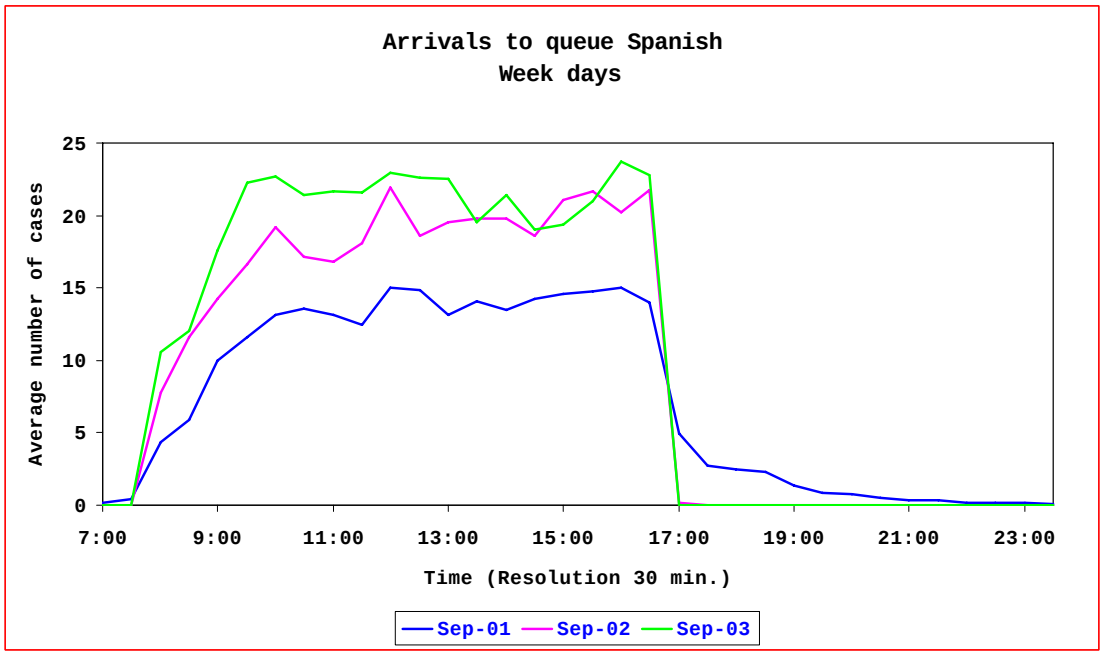


Figure 65 displays the average number of CCO calls per weekday. Note the sharp increase for December 2002. A partial explanation might be the significant decrease in the fraction of customers who receives service via VRU only, as shown in Figure 66.

Figure 65 : Average number of CCO calls per weekday

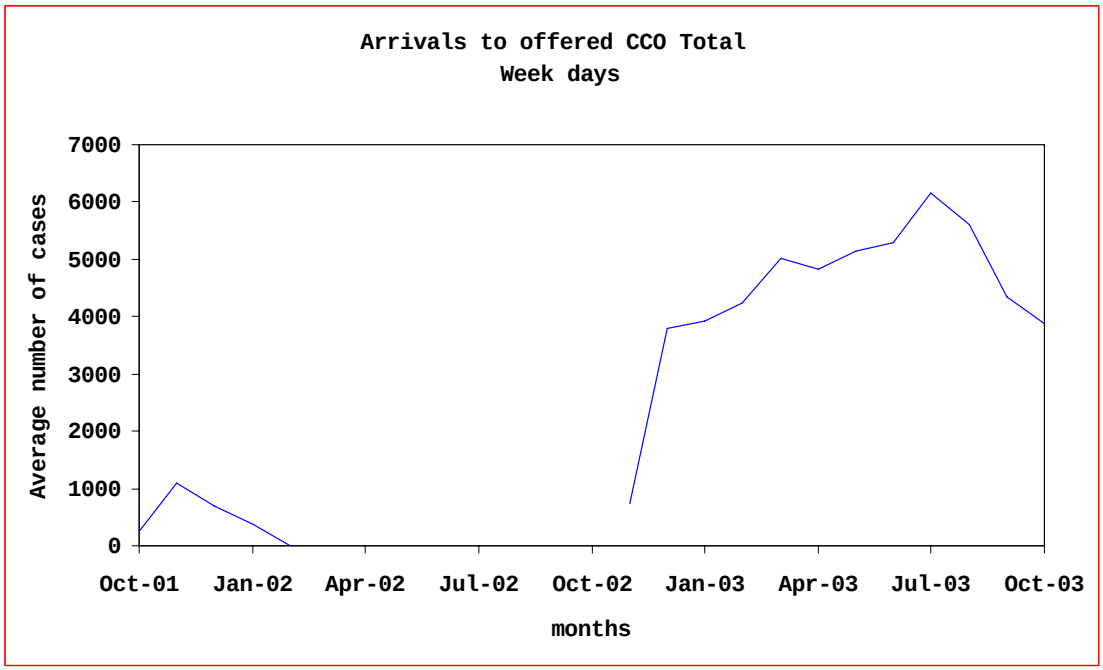
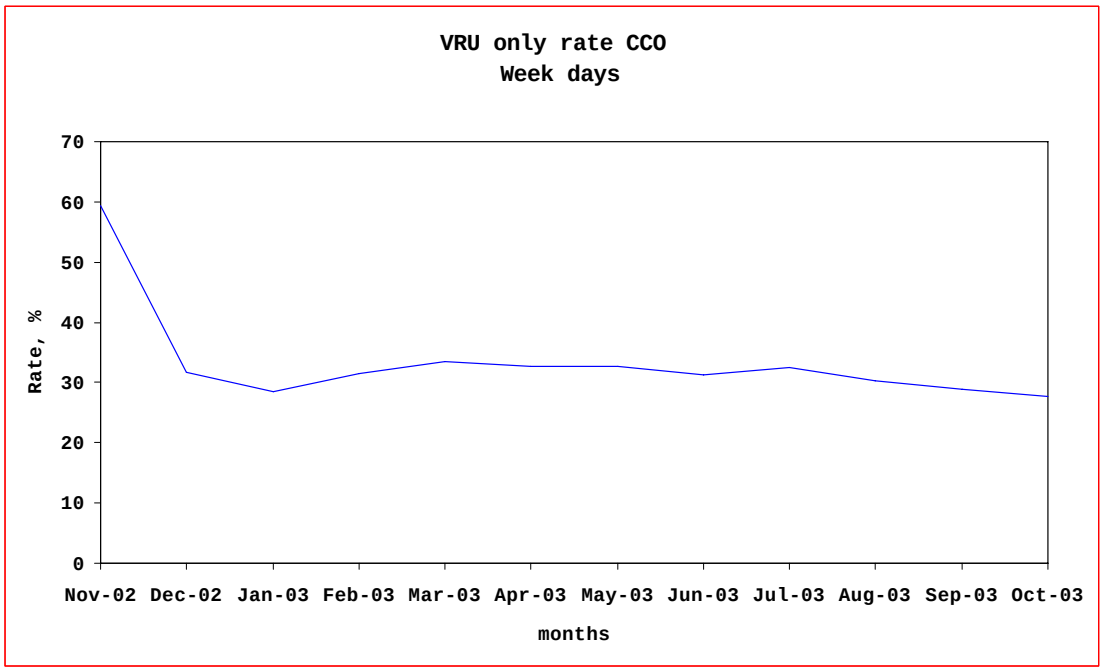


Figure 66 : Proportion of VRU-only entries for CCO calls



The Brokerage calls offered volume reach a maximum in December 2002, as shown in Figure 67. One explanation might be in the seasonal pattern: December and January are usually month with a high stock exchange transaction volume. Note also the decrease in the number of customers that completed their service in the VRU in December 2002 (see Figure 68).

Figure 67 : Average number of Brokerage calls per weekday

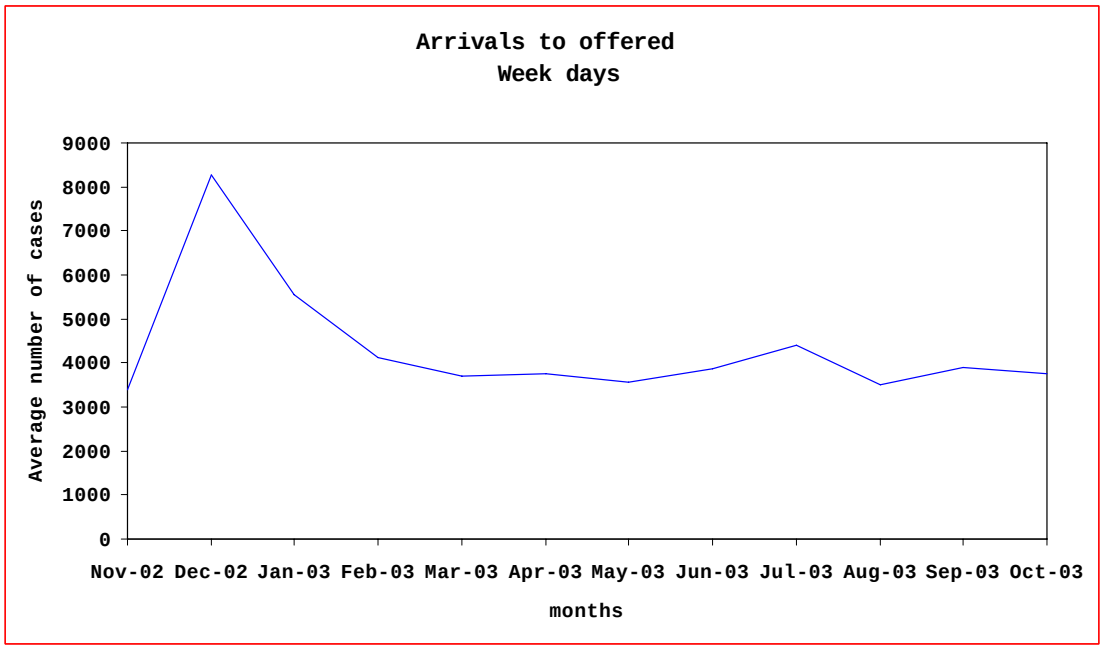


Figure 68 : Proportion of VRU-only entries for Brokerage calls

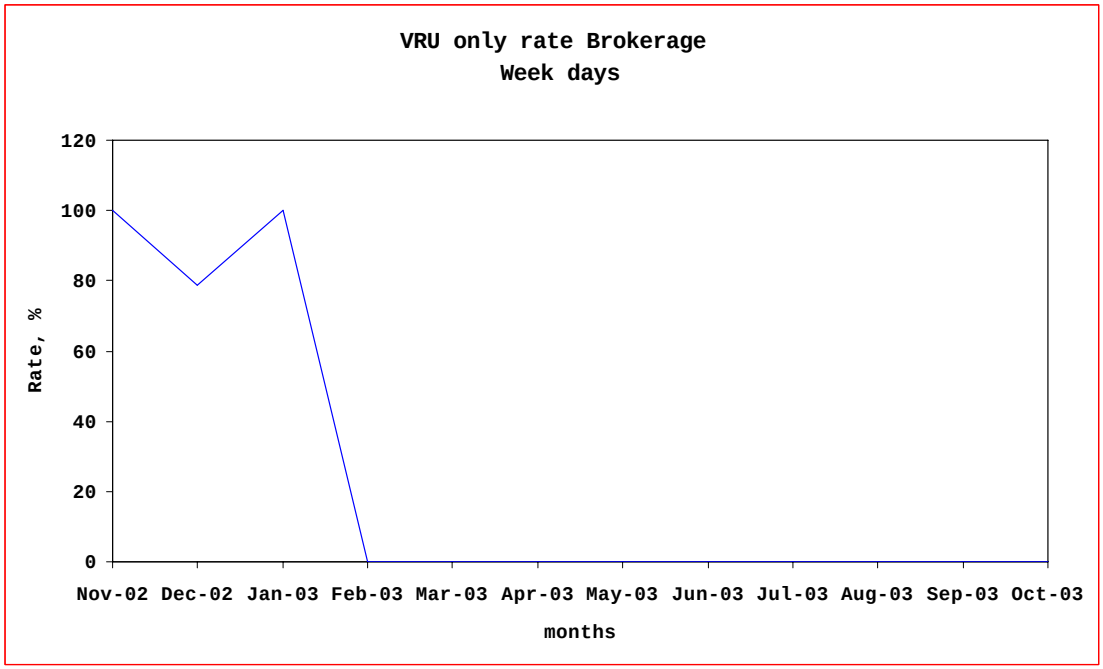


Figure 69 shows the average number of Telesales calls per weekday. We observe a sharp local decrease that occurred from October 2001 till December 2001.

Figure 69 : Average number of Telesales calls per weekday

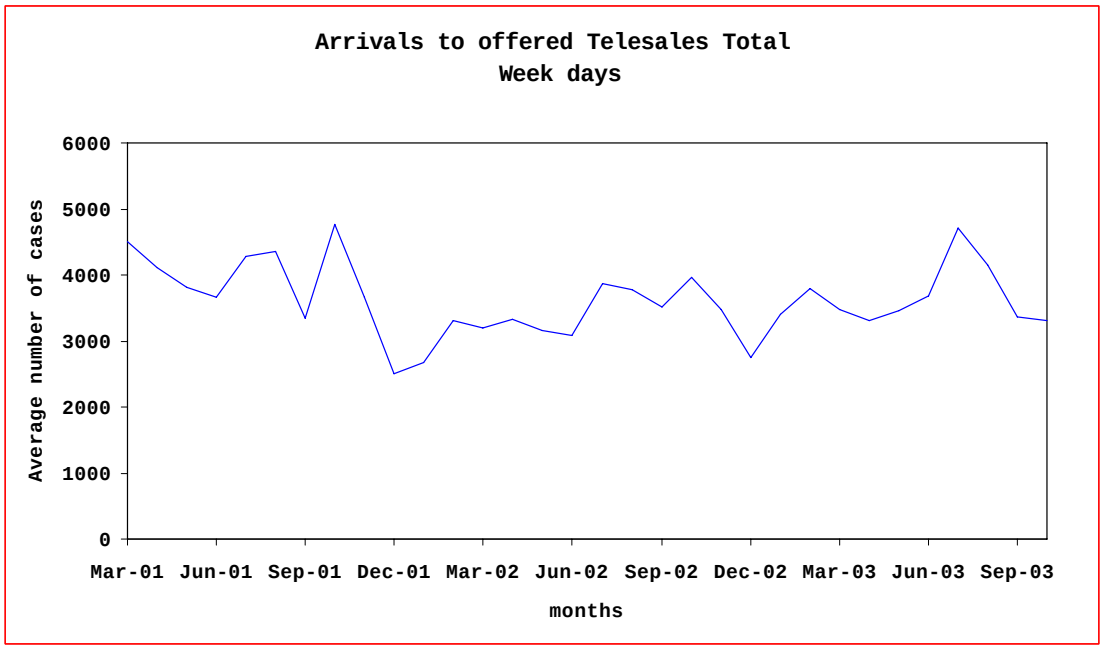


Figure 70 : Average number of Case Quality calls per weekday

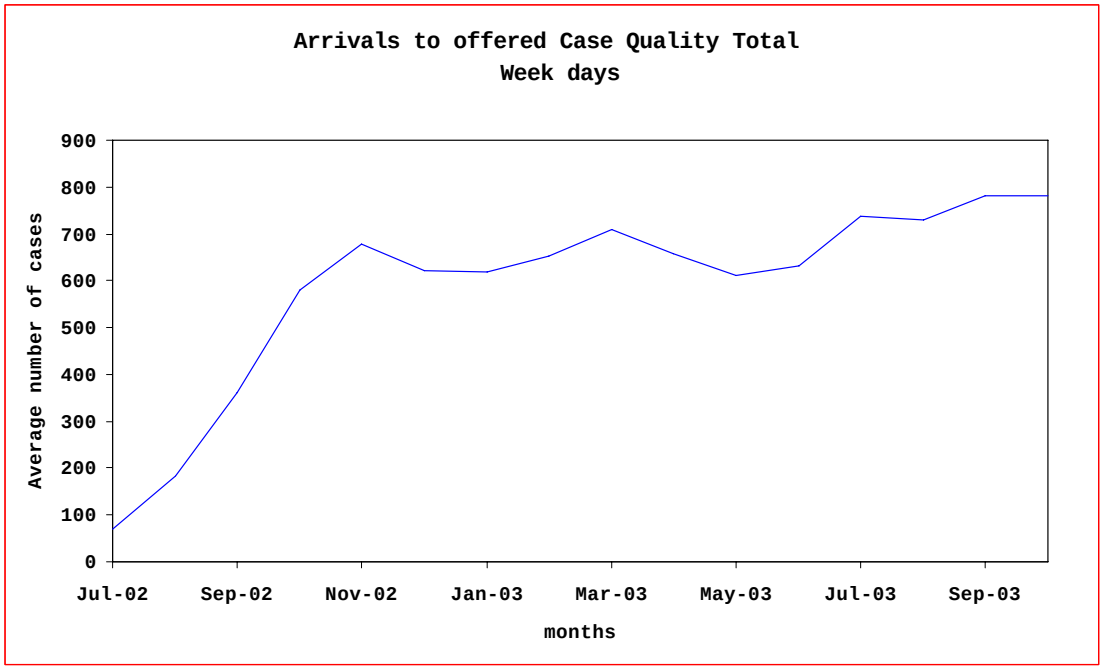


Figure 71 presents the average number of Consumer Loans calls per weekday. Note that a significant increase (decrease) in the arrivals rate usually corresponds to a decrease (increase) of the percentage of VRU-only entries (see Figure 36). For example, the average number of customers increases from June 2001 until August 2001, and fraction of customers that receive service via VRU decreases during this period.

Figure 71 : Average number of Consumer Loans calls per weekday

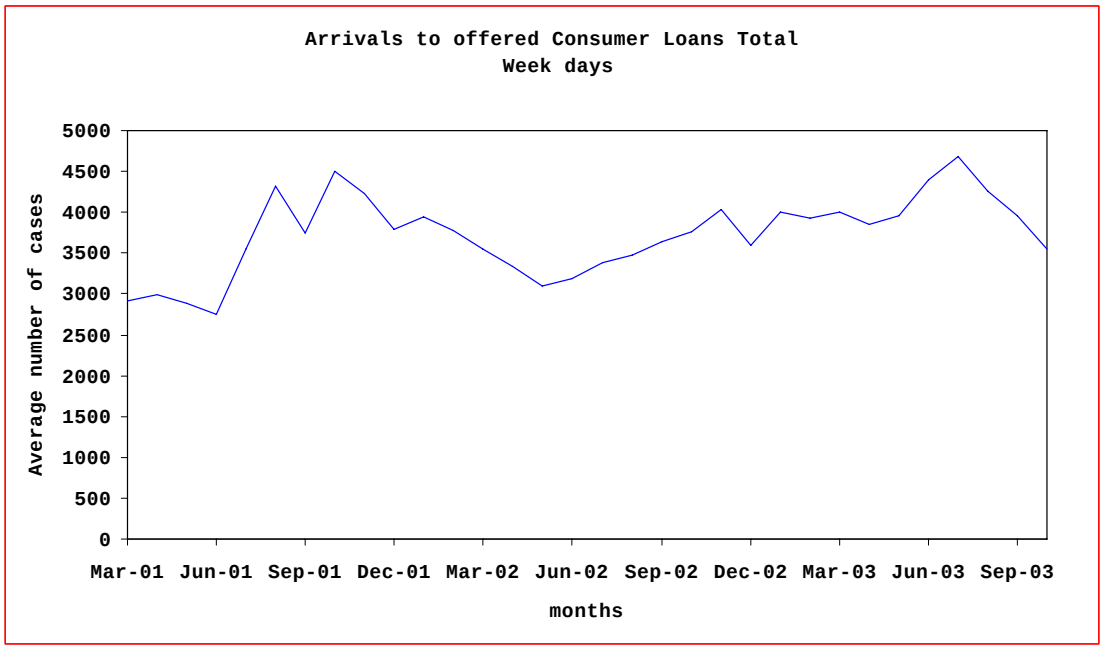
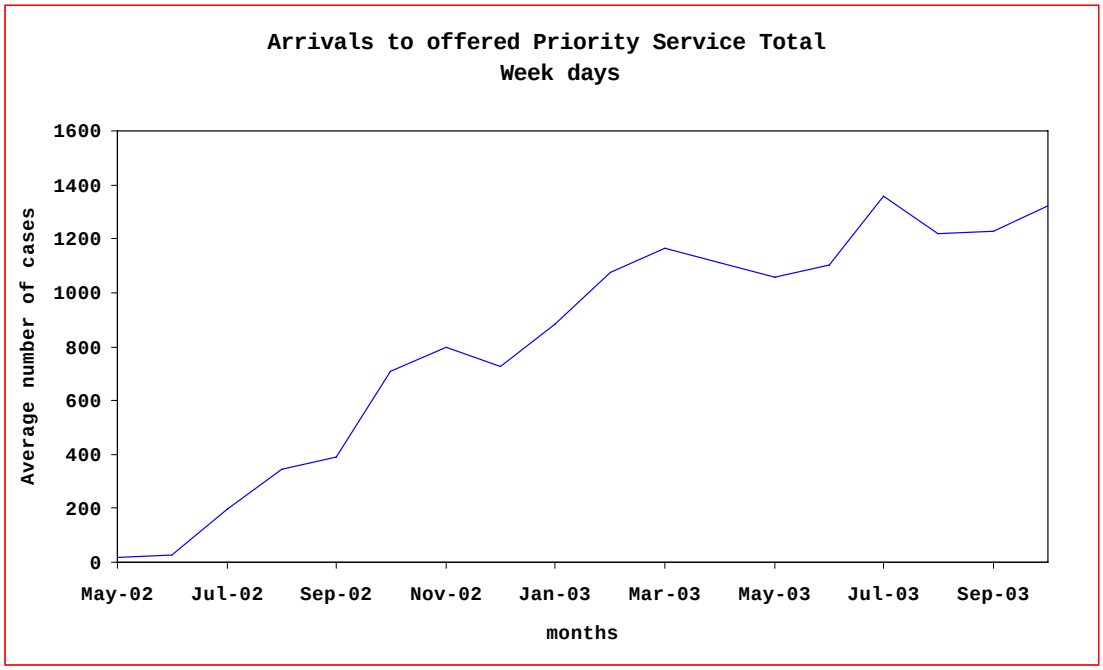


Figure 72 displays the average number of Priority Service calls per weekday. We observe an increase of Priority Service calls during the considered period.

Figure 72 : Average number of Priority Service calls per weekday



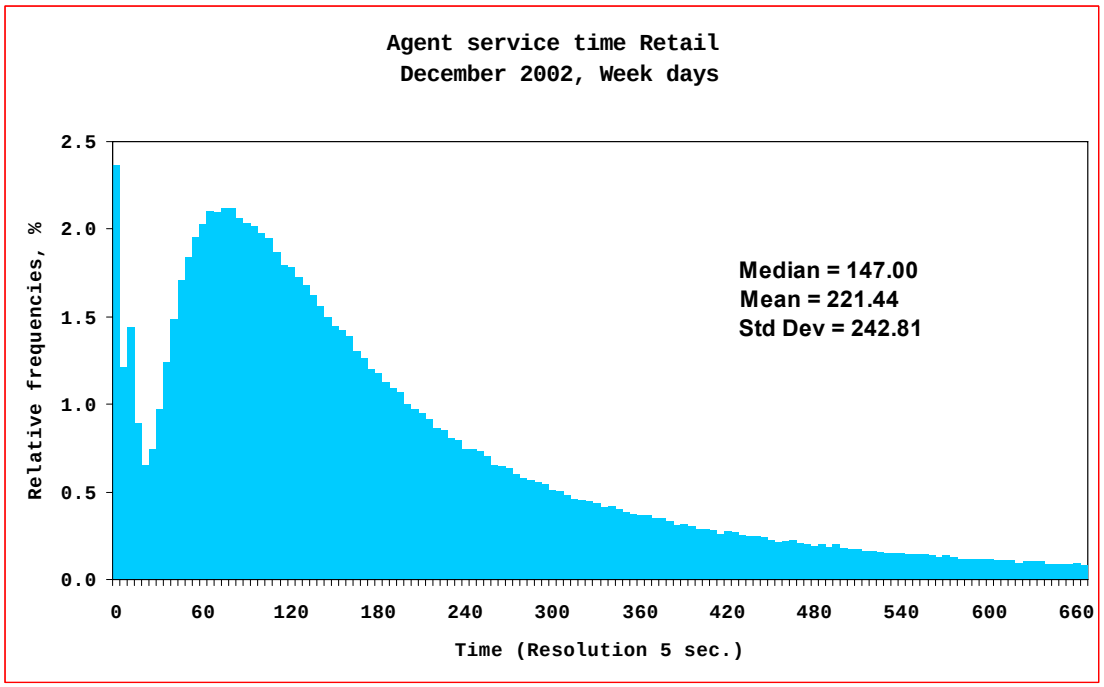
6 Agent and customer service times

Two different definitions of a service time in a call center can be considered. From a customer's point of view the service time is equal to the talk time with an agent (or several agents). From an agent's point of view it also includes additional activities connected with the service of a specific customer, e.g. the time spent on after-call work. In this section we consider both versions of service time, but we emphasize the agent service time because this is the one that should be incorporated in most mathematical models of tele-queues.

Figure 73 presents the distribution of agent service time for Retail calls. The agent service time for Retail calls has a distribution that is similar to the lognormal distribution, except for a relatively large percent of small service times. These short service times probably reflect other phenomena – and not real service. In some cases the customer (or even the agent!) may disconnect as soon as the connection is made. One therefore needs to make some judgment concerning what is the minimal time that can be legitimately considered service.

Note also that the standard deviation is smaller than the mean for Retail calls. In [1] the opposite relation was observed when considering all service times together.

Figure 73 : Distribution of agent service time for Retail calls (Dec-02)



We also consider histograms for some of the other customer types.

Figure 74 : Distribution of agent service time for Platinum calls (Dec-02)

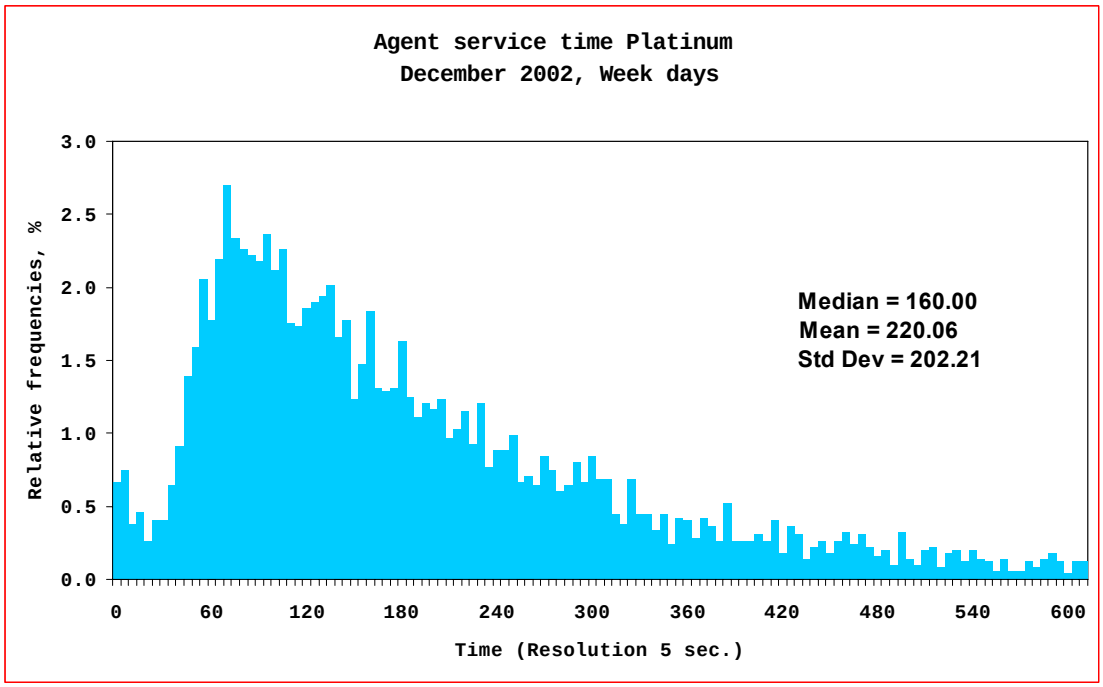
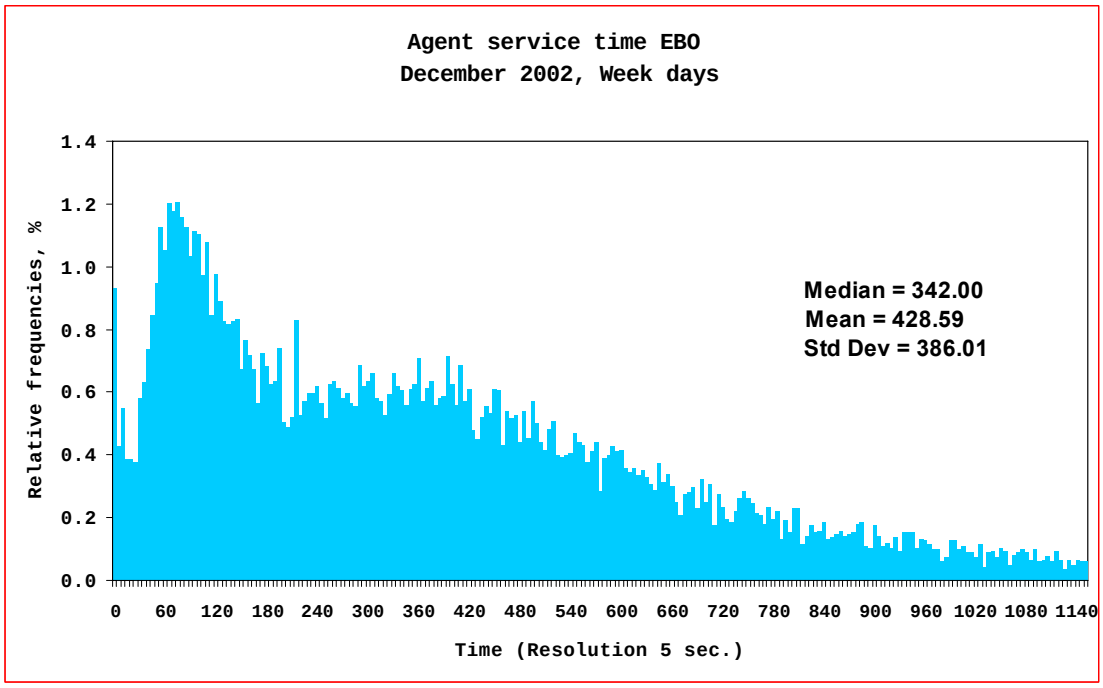


Figure 75 shows the distribution of agent service times for EBO calls. Note that EBO calls have an unusual distribution with two modes.

Figure 75 : Distribution of agent service time for EBO calls (Dec-02)



Consider now EBO service time distribution with different resolution (10 and 30 seconds). We still observe two modes for resolution 10 seconds (see Figure 76). However, for resolution 30 seconds only first mode observed. The second mode transformed to “plateau” (see Figure 77).

Figure 76 : Distribution of agent service time for EBO calls (resolution 10 sec)

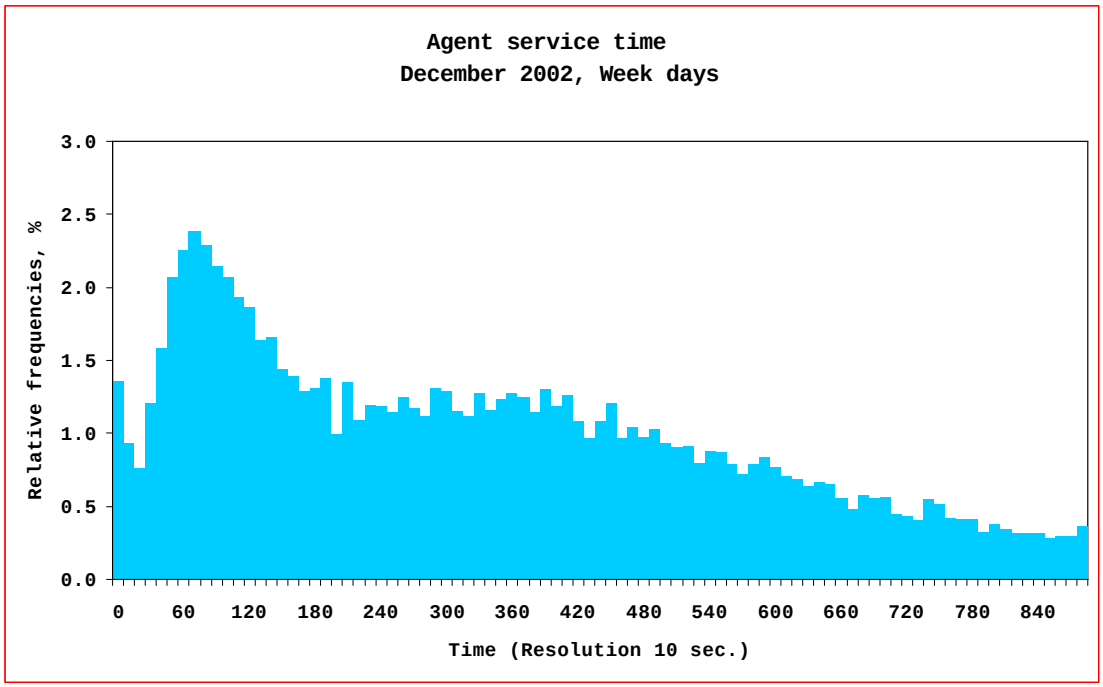


Figure 77 : Distribution of agent service time for EBO calls (resolution 30 sec)

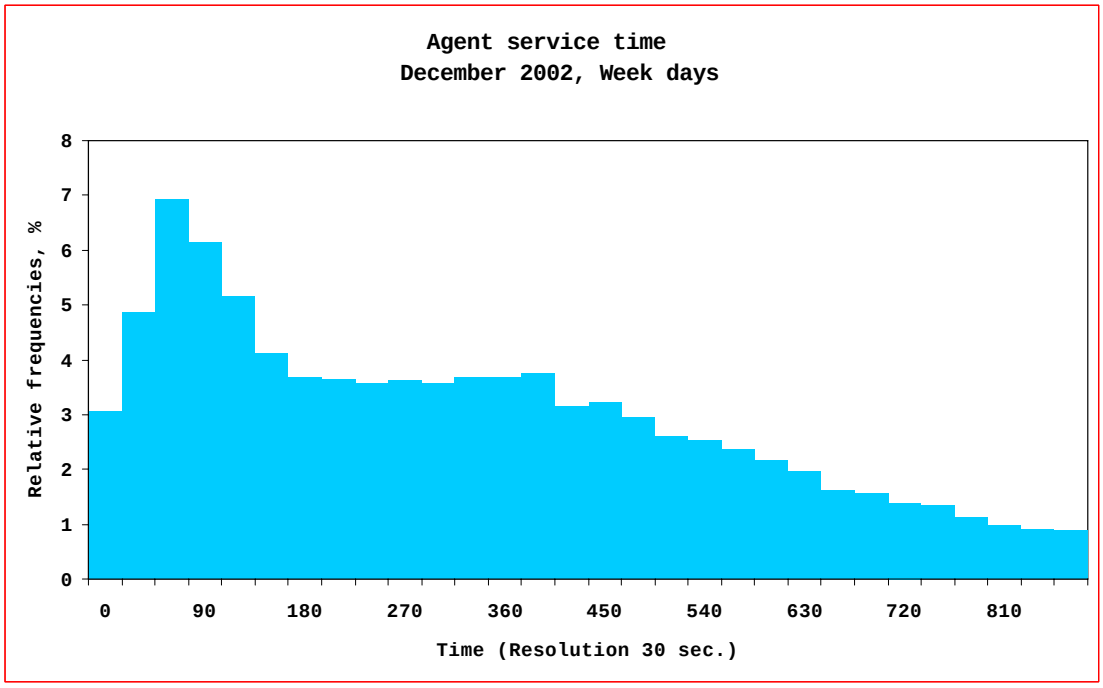


Figure 78 : Distribution of agent service time for Telesales calls (Dec-02)

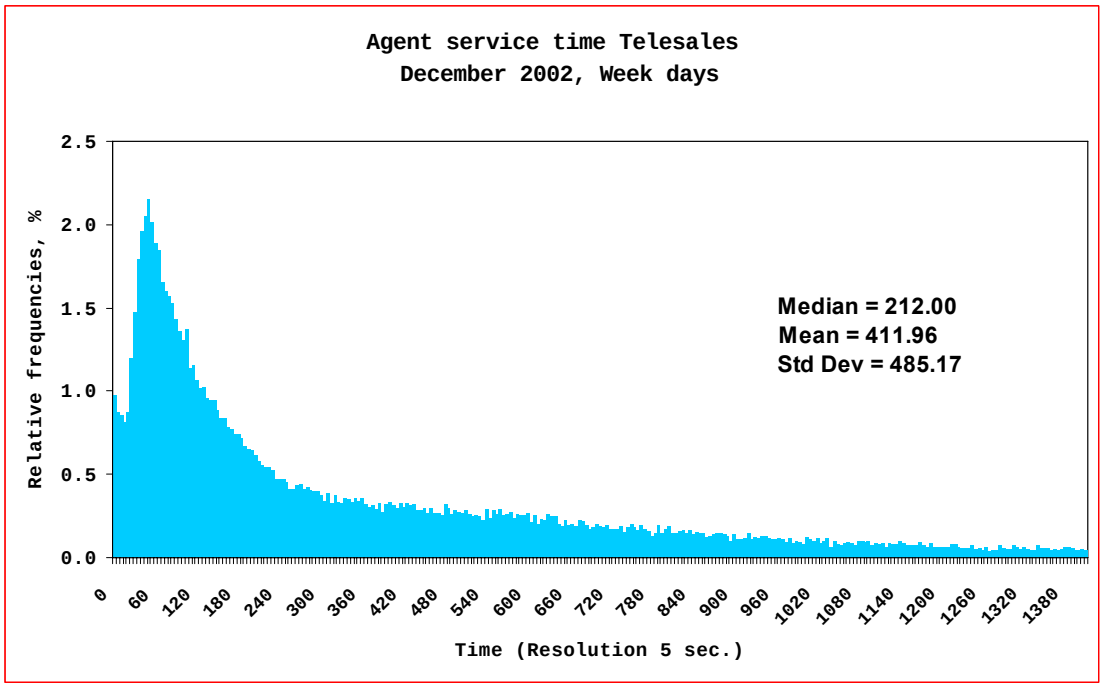


Table 13 contains service time statistics (in seconds) for Retail calls for 3 months: May-2001, May-2002 and May-2003. Note the relatively large difference between the mean in May 2001 (251 sec) and that in May 2002 (218 sec).

Table 13: Statistics for agent service time (Retail calls)

	May-01	May-02	May-03
Median	172.00	149.00	143.00
Mean	251.43	218.33	213.95
Std Dev	267.78	233.70	231.94

Figure 79- Figure 82 present the distribution of customer service time for Business calls in June 2001. We observe a significant frequency (about 4.5%) of customers that have a short service time (see Figure 79). We shall study this phenomenon in detail. Figure 80 shows distribution of customer service time for customers who were transfer to another agents. We don't observe a peak on 0. Note that about 3% of customers and about 8% of agents disconnected as soon as the connection is made (see Figure 81 and Figure 82).

Figure 79 : Distribution of customer service time for Business calls (Jun-01)

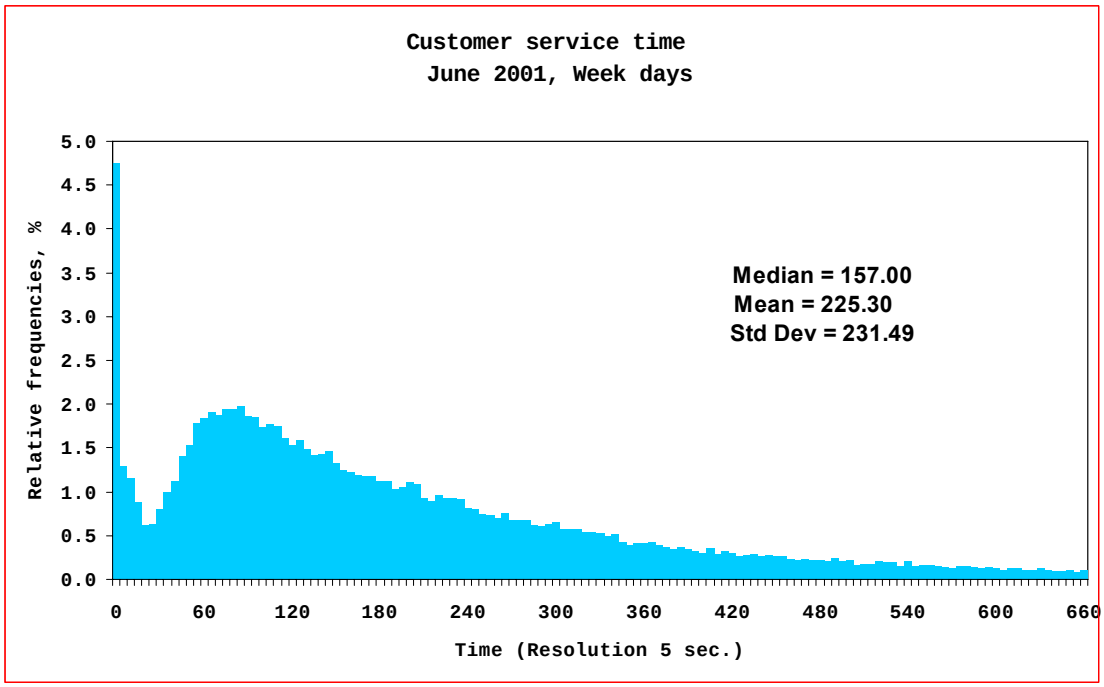


Figure 80 : Distribution of customer service time for Business calls (transfer)

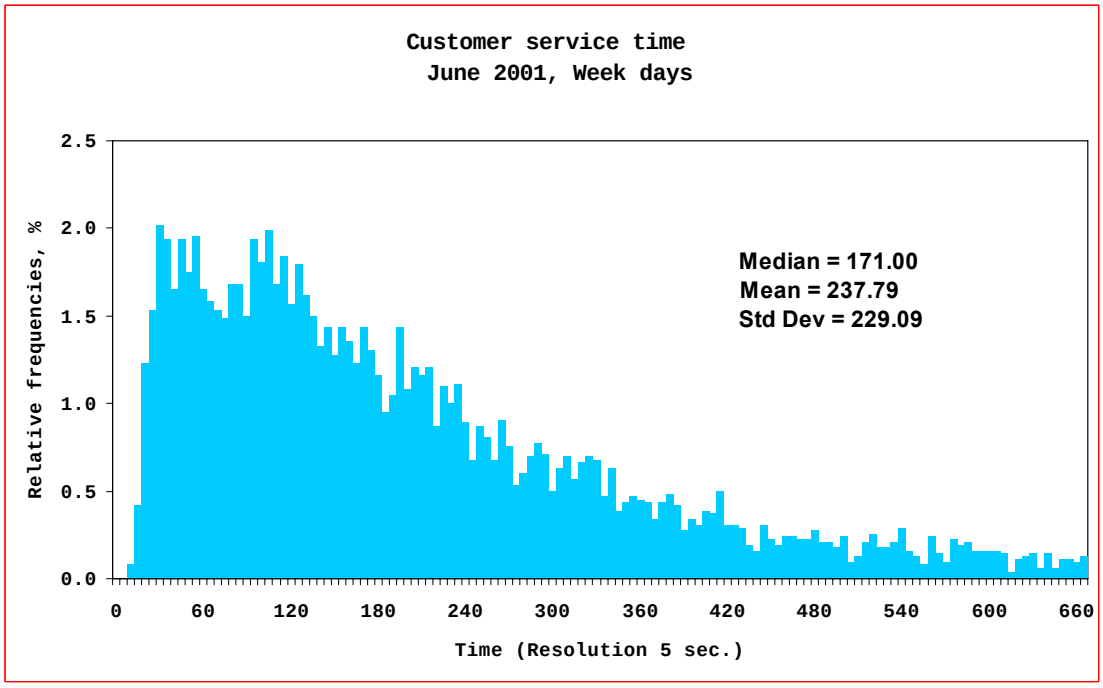


Figure 81 : Distribution of customer service time for Business calls (caller termination)

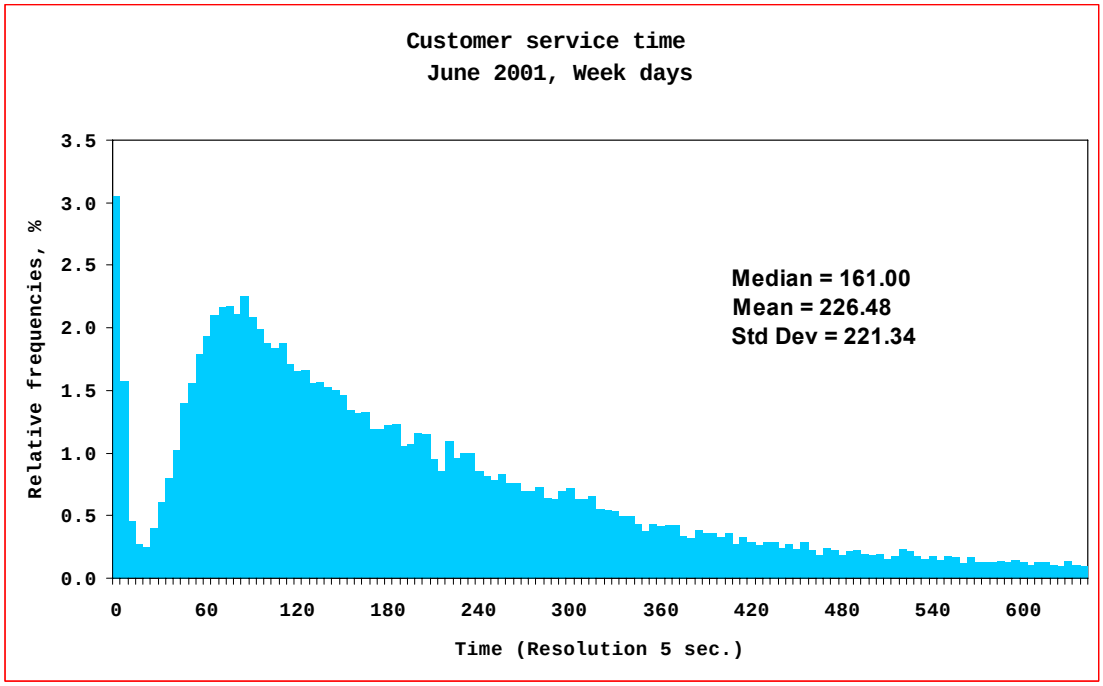


Figure 82 : Distribution of customer service time for Business calls (agent termination)

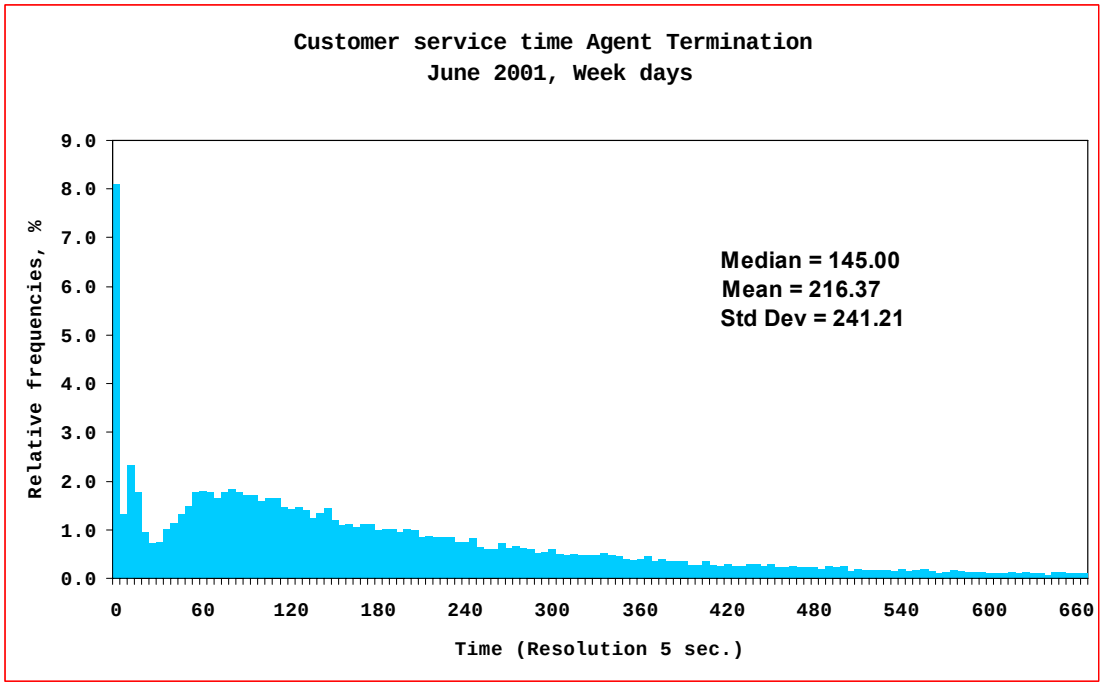
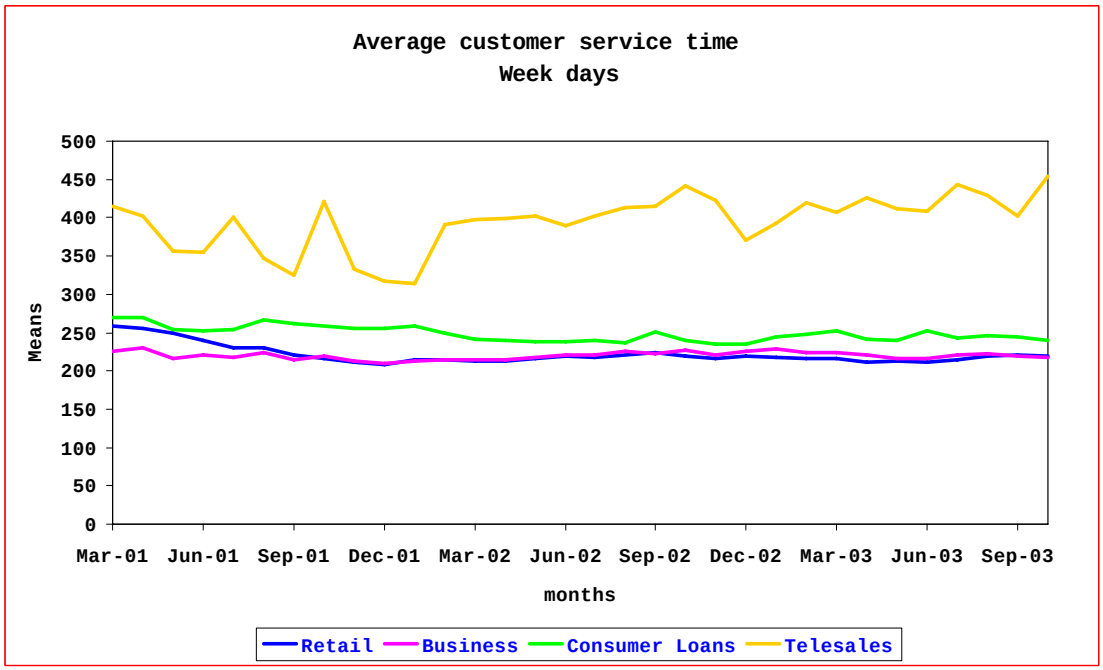


Figure 83 describes the dynamics of average customer service time for some prevalent customer types. We observe that service time of Telesales calls is largest and unstable (yellow line). Note that average service time of Retail decrease until December 2001 and, then, stabilizes (blue line). Average service time of Business calls is stable during all considered period (violet line). Finally, average service time of Consumer Loans calls decrease until August 2002 (green line).

Figure 83 : Average customer service time for some customer types



Consider the average customer service time for Retail, Business and Telesales calls, for three months: May 2001, May 2002 and May 2003.

Figure 84 shows the dynamics of average customer service times for Retail calls. We observe that average service time in May 2001 is larger than in May 2002 and May 2003 in daily hours. Note that the average service time is unstable during night hours. Average service time increases after 7am and it reaches maximum between 9am and 3:30pm.

Figure 84 : Dynamics of average customer service time for Retail calls

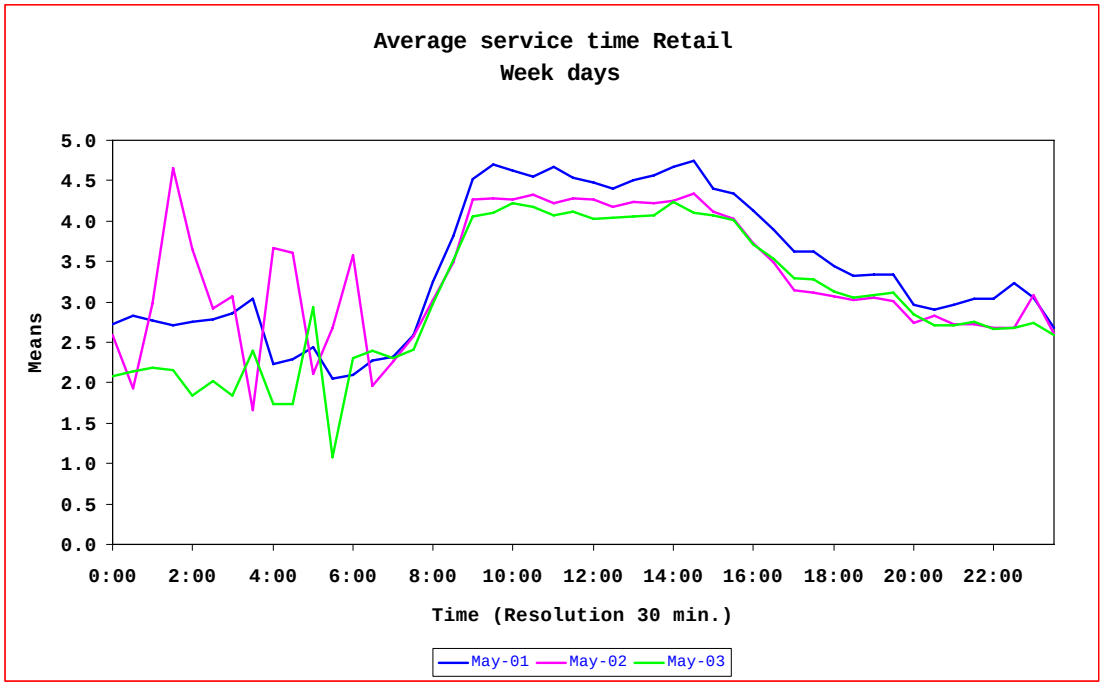


Figure 85 shows the dynamics of average customer service time for Business calls. Consider daylight hours (say, from 6am): average service time increases after 6am till 9:30am and remains quite stable till 16:30 in May 2001 and May 2002.

Figure 85 : Dynamics of average customer service time for Business calls

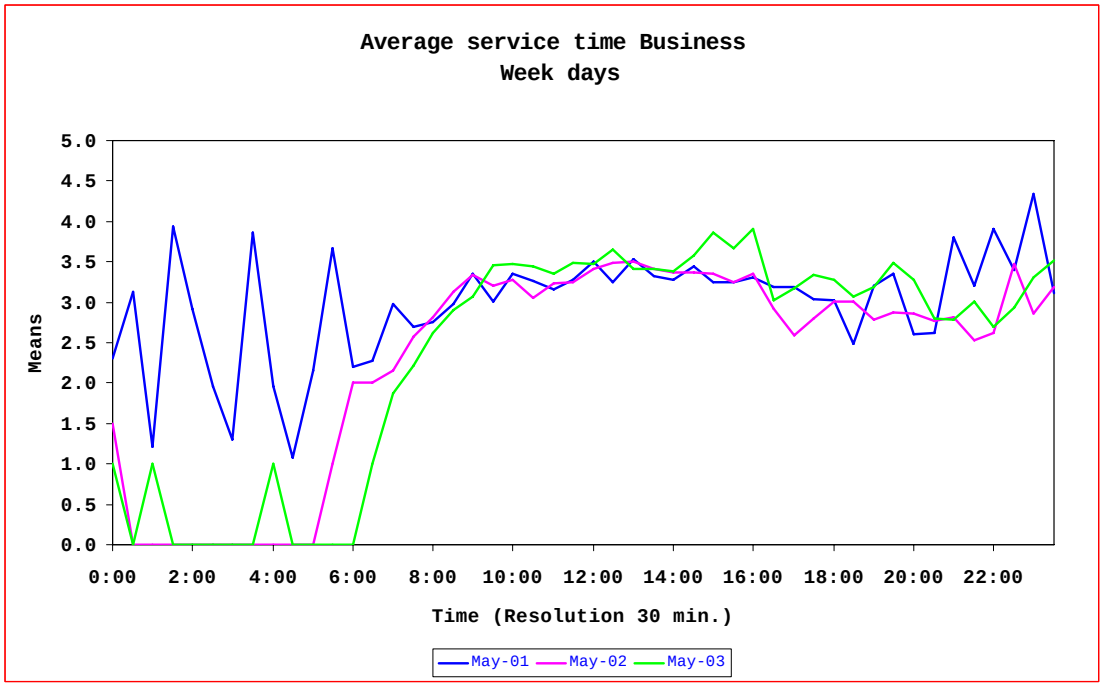
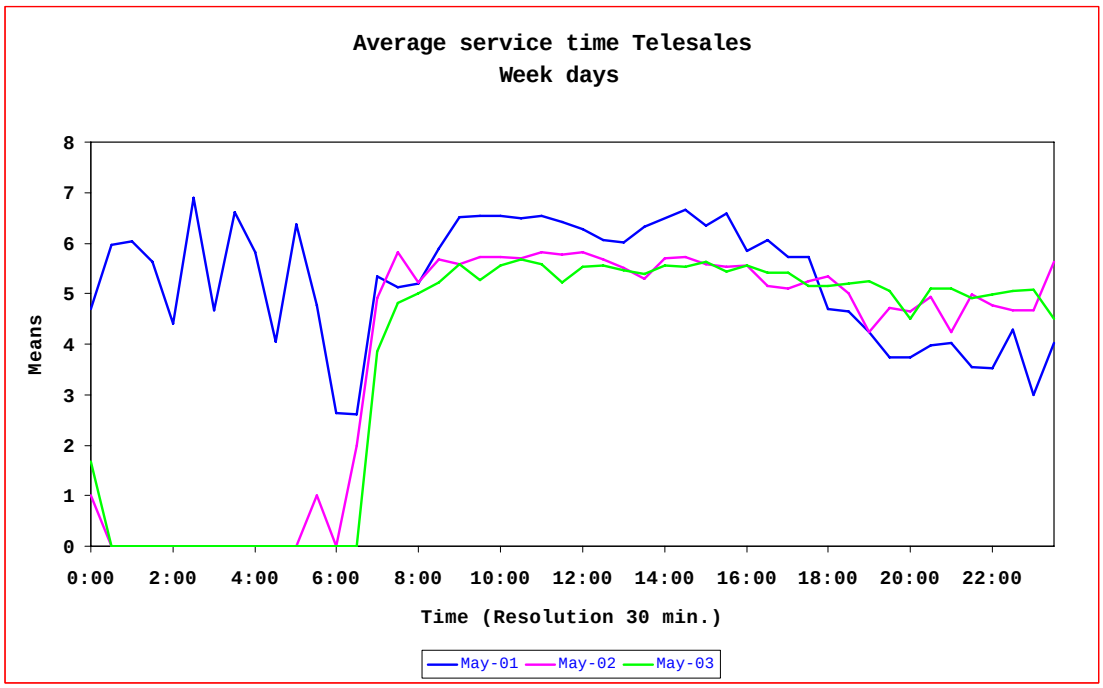


Figure 86 shows the dynamics of average customer service time for Telesales calls. We observe increase from 6am till 8:30am and decrease after 5:30pm. Average service time in May 2001 is larger than in May 2002 and May 2003 between 8:30am and 6pm.

Figure 86 : Dynamics of average customer service time for Telesales calls



Now consider the difference between agent and customer monthly service times for all types of customers (specifically May, 2001 and December, 2002). Table 14 gives summary statistics for service times. Note that difference between agent and customer service times is largest for Telesales (36.40 sec), Retail (32.14 sec) and EBO (32.07 sec) customers. On the other hand, it is a zero for Case Quality, Priority Service, AST, CCO, Brokerage, Mortgage and BPS customer types, suggesting that there is no after-call work for these types of calls. We also observe the following phenomenon: for Online Banking and EBO calls, expected customer service time is larger than expected agent service time. One explanation for EBO calls might be that sometimes an agent connects a customer to a consulting agent during his service. In these cases the service time of the consulting agent is less than the customer service time. Note also that for Online Banking calls we observe this phenomenon in April, 2001 and May, 2001 only.

Table 14: Summary statistics for service time

Types	Mean of Agent Service Time	Mean of Customer Service Time	Difference: Agent - Customers
Retail, May-01	251.43	219.29	32.14
Premier, May-01	306.39	299.53	6.86
Busibess, May-01	226.24	219.75	6.49
Platinum, May-01	247.55	234.8	12.75
Consumer Loans, May-01	257.34	254.83	2.51
Online Banking, May-01	430.47	431.77	-1.30
EBO, May-01	457.65	489.72	-32.07
Telesales, May-01	391.53	355.13	36.40
Spanish, May-01	274.34	272.68	1.66
Case Quality, Dec-02	337.79	337.79	0.00
Priority Service, Dec-02	358.19	358.19	0.00
AST, Dec-02	299.94	299.94	0.00
CCO, Dec-02	261.28	261.28	0.00
Acquired Bank , May-01	198.12	195.41	2.71
Brokerage, Dec-02	249.68	249.68	0.00
Mortgage, Dec-02	184.09	184.09	0.00
BPS, Dec-02	234.38	234.38	0.00

7 Waiting Time and Abandonment

After the VRU customers that need to talk with an agent are moved to the tele-queue. If there are no agents immediately available (we assume that customers who waited 0 or 1 second receive service immediately) a customer has to wait. Two major outcomes of wait are possible: a customer can reach an agent or leave the system without being served.
We have a following statistics about waiting time in our system:

- Overall waiting time (including 0 or 1 for non-waiters)
- Waiting time (for customers who wait a positive time – that is 2 or more seconds)
- Handled waiting time (for customers who reach an agent)
- Unhandled waiting time (for customers who leave the system)
- Service without waiting (for customers who wait 0 or 1 second)

In this section we consider: waiting times (average and distribution) in Subsection 7.1, service without waiting (customers who waited 0 or 1 second) in Subsection 7.2 and, finally, abandonment in Subsection 7.3.

7.1 Waiting Times

Table 15 summarizes the main waiting time characteristics for different customer types. Specifically, for each type it includes:

- Overall waiting time (median, mean and standard deviation)
- Average waiting time of customers with a positive wait
- Fraction of customers with a positive wait

Recall that positive waiting time means 2 or more seconds of wait.

Table 15: Summary statistics for waiting time

Type	Wait time (all)			Mean of wait time(waiting)	% waiting (>1 Sec)
	Median	Mean	Std Dev		
Retail, May-01	1.00	3.64	16.58	19.67	15.33
Premier, May-01	2.00	4.43	23.31	6.44	64.10
Business, May-01	1.00	4.85	33.42	56.66	7.47
Platinum, May-01	1.00	5.40	21.29	31.27	15.41
Consumer Loans, May-01	1.00	6.45	38.91	41.34	14.46
Online Banking, May-01	1.00	2.44	19.70	27.07	7.28
EBO, May-01	1.00	18.35	70.85	47.03	38.48
Telesales, May-01	0.00	6.64	28.01	37.68	17.25
Spanish, May-01	8.00	54.37	93.50	103.83	52.07
Case Quality, Dec-02	1.00	13.27	24.35	30.94	41.68
Priority Service, Dec-02	12.00	26.74	36.72	39.84	66.58
AST, Dec-02	1.00	21.39	66.09	76.37	27.28
CCO, Dec-02	22.00	162.66	283.51	216.43	75.08
Acquired Bank, May-01	1.00	19.36	58.19	61.10	30.93
Brokerage, Dec-02	66.00	210.87	89.30	268.90	78.38
Mortgage, Dec-02	5.00	22.20	57.68	42.02	52.10
BPS, Dec-02	1.00	28.96	4.58	89.80	31.78

Main observations:

- High service level (average wait < 10 second) is observed for the majority of customer types.
- On the other hand, the expected wait of Premier calls (4.43 sec) is larger than the one of Retail calls (3.64 sec). Also, the expected wait of Platinum calls (5.40 sec) is larger than the one of Business calls (4.85 sec). Recall that Premier is the high priority version of Retail, and Platinum is high priority for Business.
- In some cases we observe interesting relationships between expected wait time and percentage of waiting (for example, Premier, Business, and Online Banking).

Table 16: Waiting time per month, 2001

Date	Mar-01	Apr-01	May-01	Jun-01	Jul-01	Aug-01	Sep-01	Oct-01	Nov-01	Dec-01
Retail	6.7	5.0	3.7	2.1	2.9	4.7	3.5	2.8	3.0	7.4
Premier	5.2	5.9	4.7	3.4	3.0	4.0	4.0	3.6	3.3	5.2
Business	1.8	4.3	5.0	2.4	2.2	2.9	3.7	5.8	4.2	8.0
Platinum	6.9	6.7	5.6	4.3	3.5	5.1	6.3	5.5	6.8	8.7
Consumer Loans	6.1	7.1	7.0	3.6	12.8	20.3	18.6	45.5	24.1	17.5
Online Banking	2.8	2.6	2.4	1.9	4.6	3.2	2.6	3.5	2.4	4.0
EBO	68.8	35.0	23.6	12.6	18.0	23.6	19.8	11.9	13.2	23.7
Telesales	25.1	20.5	6.8	10.2	14.8	6.9	3.7	115.9	3.9	4.0
Spanish	34.6	47.4	53.8	43.0	103.7	162.6	20.2	21.8	19.9	19.7
CCO	0.0	0.0	0.0	0.0	0.0	0.0	0.0	327.9	15.3	78.8
Acquired Bank	22.9	10.4	19.4	28.6	46.0	0.0	0.0	0.0	0.0	0.0

Table 17 : Waiting time per month, 2002

Date	Jan 02	Feb 02	Mar 02	Apr 02	May 02	Jun 02	Jul 02	Aug 02	Sep 02	Oct 02	Nov 02	Dec 02
Retail	11.4	9.8	10.1	3.7	3.7	6.4	9.7	8.1	12.2	7.2	5.6	5.3
Premier	5.8	4.6	7.9	3.2	3.3	4.6	6.9	6.6	9.1	6.5	7.5	7.2
Business	8.7	7.1	6.0	6.8	7.5	10.3	18.2	14.9	20.0	10.0	14.0	13.6
Platinum	8.8	7.6	5.7	4.2	5.1	5.6	8.6	8.4	11.3	6.7	8.3	9.0
Consumer Loan	14.4	14.1	8.3	6.2	7.5	9.6	15.2	13.9	14.4	11.0	24.4	14.0
Online Banking	5.6	6.8	9.7	2.9	4.0	4.7	10.8	11.0	16.6	14.3	13.7	16.3
EBO	18.9	17.2	15.6	10.0	12.3	22.3	15.2	16.9	33.1	26.6	32.4	26.4
Telesales	3.5	6.3	4.8	3.6	2.6	2.0	13.0	9.2	13.3	15.8	12.9	2.9
Spanish	13.8	20.1	17.7	24.5	24.0	32.4	11.3	10.3	7.0	4.8	4.3	4.9
Case Quality							8.7	12.3	18.8	16.2	18.2	13.5
Priority Service					18.7	17.8	24.6	32.7	35.7	29.8	30.5	27.7
AST											40.9	21.4
CCO	60.7	9.2									229.1	167.4
Brokerage											81.8	232.3
Mortgage											58.4	25.7
BPS											8.1	30.4

Table 18 : Waiting time per month, 2003

Type	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03
Retail	6.9	11.8	8.5	7.8	8.0	8.6	9.5	9.8	8.8	4.4
Premier	7.6	7.3	4.5	3.9	5.2	5.8	8.5	12.4	11.6	10.1
Business	12.7	10.2	10.0	6.7	9.3	10.4	12.8	9.8	10.9	7.7
Platinum	8.1	6.6	7.8	5.5	7.2	7.5	15.0	7.8	10.0	8.4
Consumer Loan	26.9	22.7	6.9	3.7	7.5	12.2	11.3	10.6	5.7	5.0
Online Banking	11.8	9.7	8.5	4.0	3.6	4.9	15.3	13.7	20.1	12.1
EBO	22.1	33.6	17.3	19.0	21.5	21.0	13.6	12.2	10.4	8.8
Telesales	7.8	10.9	6.7	4.6	8.8	10.3	23.0	16.1	3.6	5.7
Spanish	6.7	21.4	9.6	15.2	13.0	6.1	8.4	5.6	4.7	2.7
Case Quality	13.4	13.7	13.6	12.2	12.4	11.8	17.1	19.3	22.2	15.2
Priority Service	25.6	28.1	26.1	24.7	25.5	25.7	28.9	28.9	28.3	26.8
AST	7.8	6.2	3.7	5.6	11.7	16.3	19.6	27.1	15.9	10.8
CCO	14.7	11.1	8.5	8.8	8.6	6.4	26.1	27.4	12.3	7.2
Brokerage	88.4	20.3	17.9	21.9	18.5	25.8	50.5	23.7	31.2	14.0
Mortgage	28.4									
BPS	27.2	43.3	70.3	49.8	47.9	101.1	116.0	81.0	167.8	270.9

Now consider the average waiting time over the whole period (from March, 2001 until October, 2003) for two pairs of call types: Retail – Platinum and Business – Premier.

Figure 87 shows average waiting time for Retail and Premier customer types. We see that in spite of the higher priority, the average waiting time for Premier calls is larger than for Retail calls during some periods (March 2001 – July 2001, September 2001 – November 2001, November 2002 – January 2003 and August 2003 – October 2003).

Figure 87 : Average waiting time for Retail and Premier calls

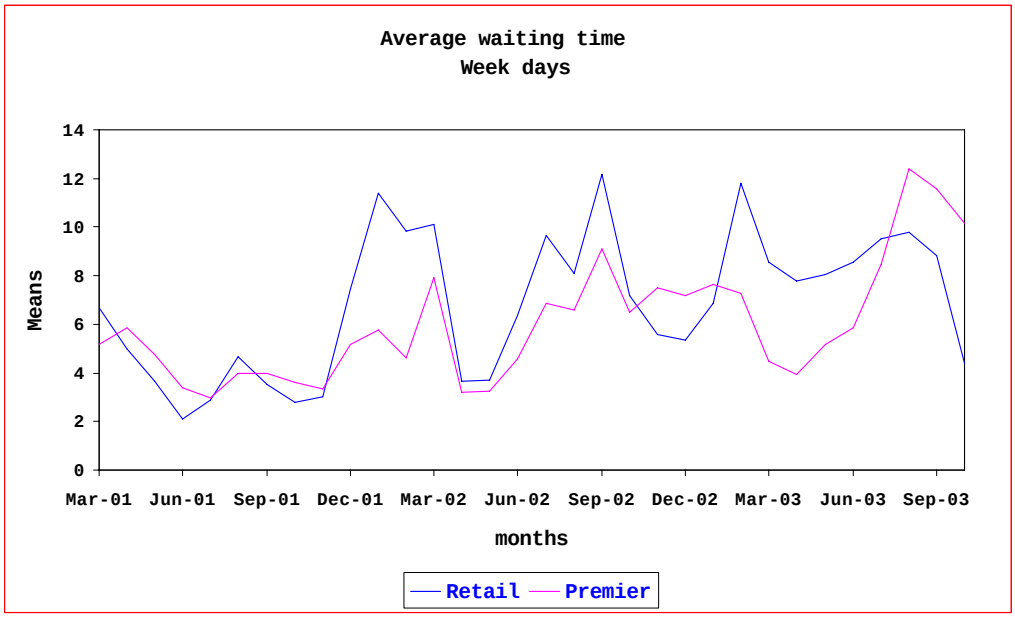
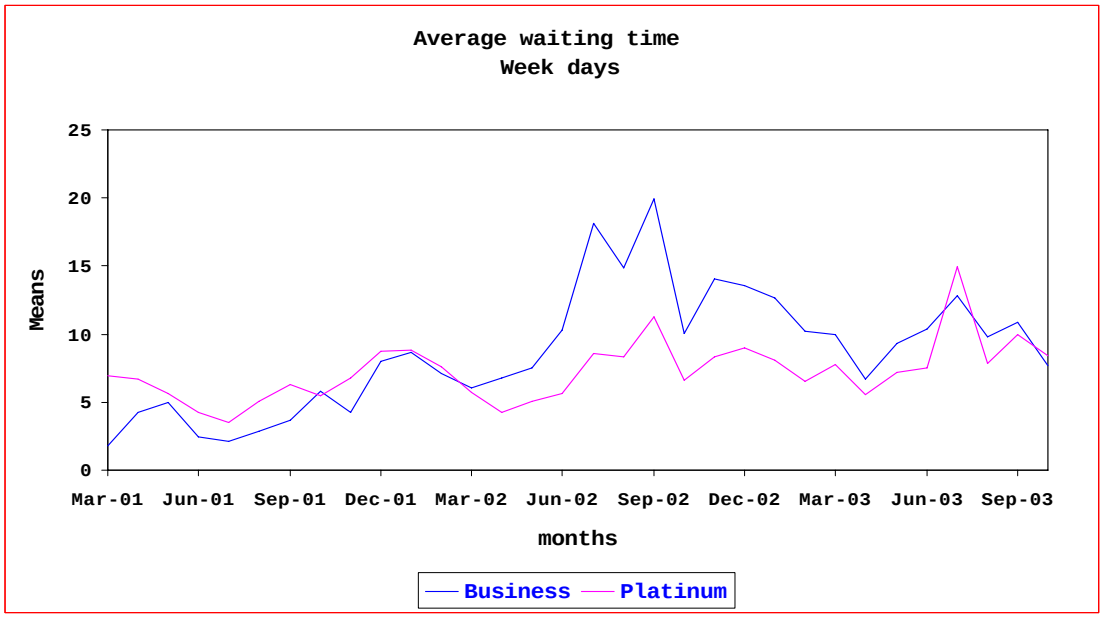


Figure 88 presents average waiting time for Business and Platinum customer types. Note that, again, the waiting time of higher priority customers (Platinum) are larger than the expected wait of regular customers during some periods (March 2001 – September 2001, November 2001 – February 2002 and July 2003).

Figure 88 : Average waiting time for Business and Platinum calls



We now consider the distribution of positive waiting times for different types of customers.

Figure 89 shows distribution of Retail waiting times. There are four separate call centers. Each customer waits in queue in a specific call center and after 10-11 seconds of waiting he or she is moved to the joint inter-queue. Therefore, we observe the mode which occurs at the 11^{th} second (17.65%).

Figure 89 : Distribution of wait time (waiting) for Retail calls (May-02)

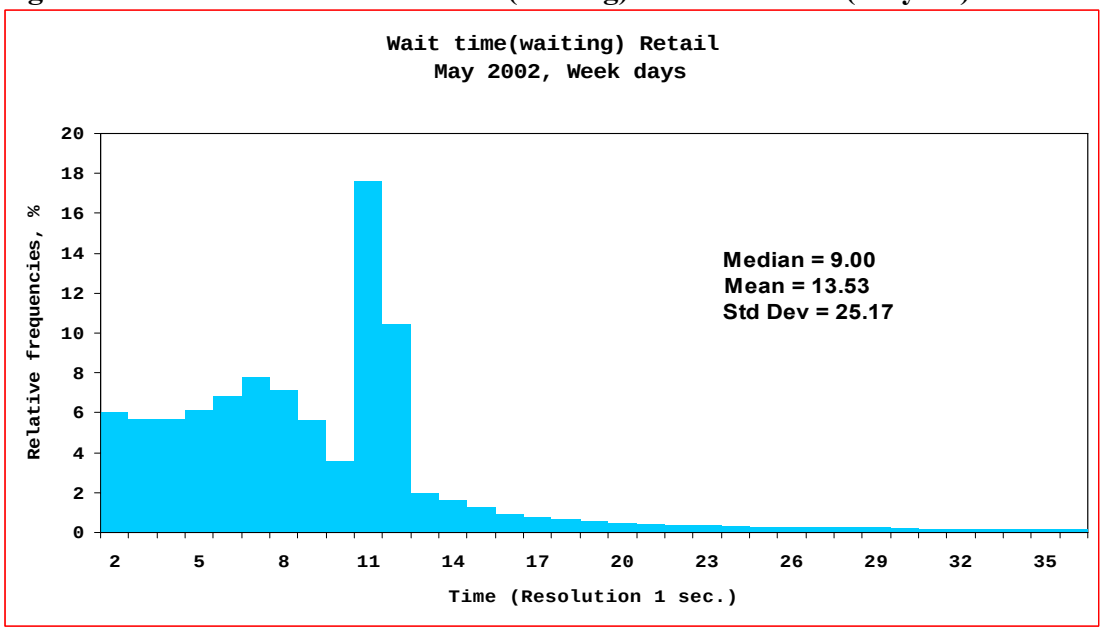


Figure 90 shows the distribution of Telesales waiting times. Note some modes at the 2nd (largest), 5th, 12th, 14th, 28th and 48th seconds.

Figure 90 : Distribution of wait time (waiting) for Telesales calls (May-01)

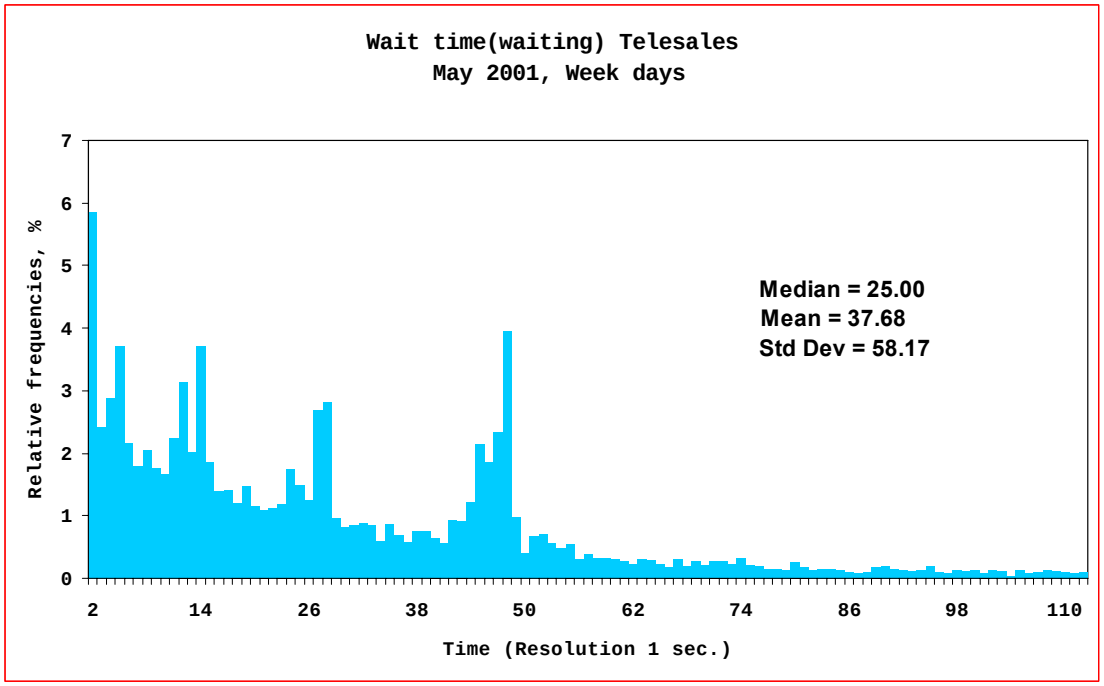


Figure 91 presents the distribution of Telesales unhandled wait times (that is, the waiting time for those customers who leave the system). We observe two modes: at zero seconds and at 45 seconds. Note that the fraction of unhandled callers who wait between 2 and 40 seconds is relatively small.

Figure 91 : Distribution of unhandled for Telesales calls (May-01)

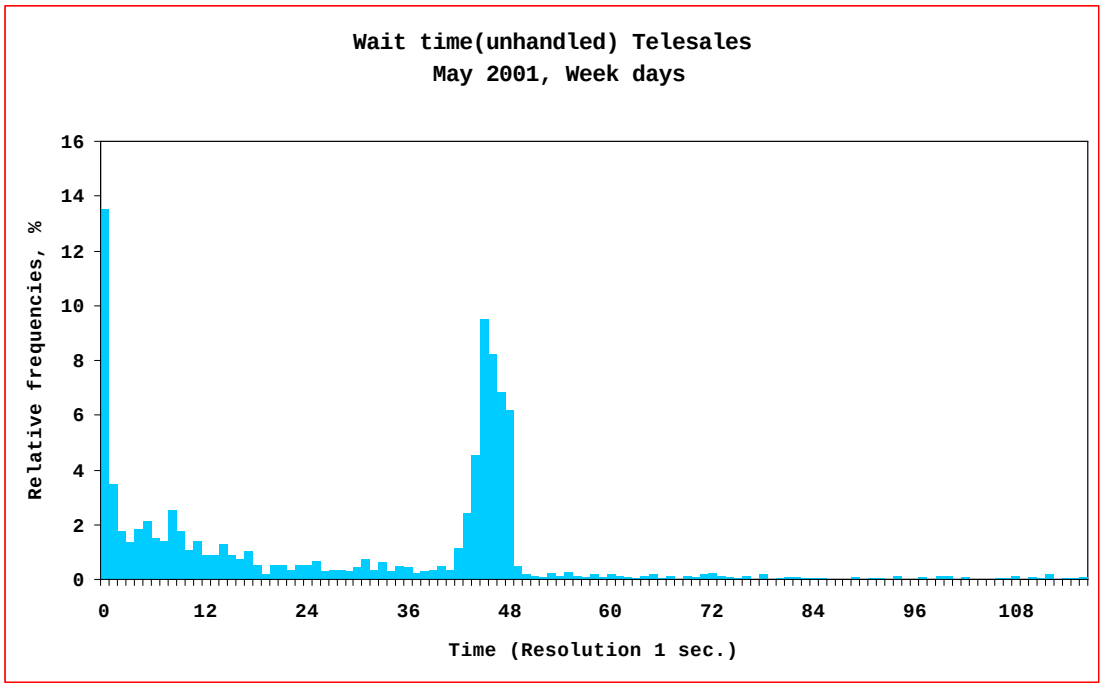


Figure 92 shows the distribution of Brokerage waiting times for callers who have to wait at all. Note that there are some (decreasing in size) modes at 31 sec (largest), 162 sec, 303 sec and 423 sec. The most likely correspond to responses to announcements or messages delivered while waiting.

Figure 92 : Distribution of wait time (waiting) for Brokerage calls (Dec-02)

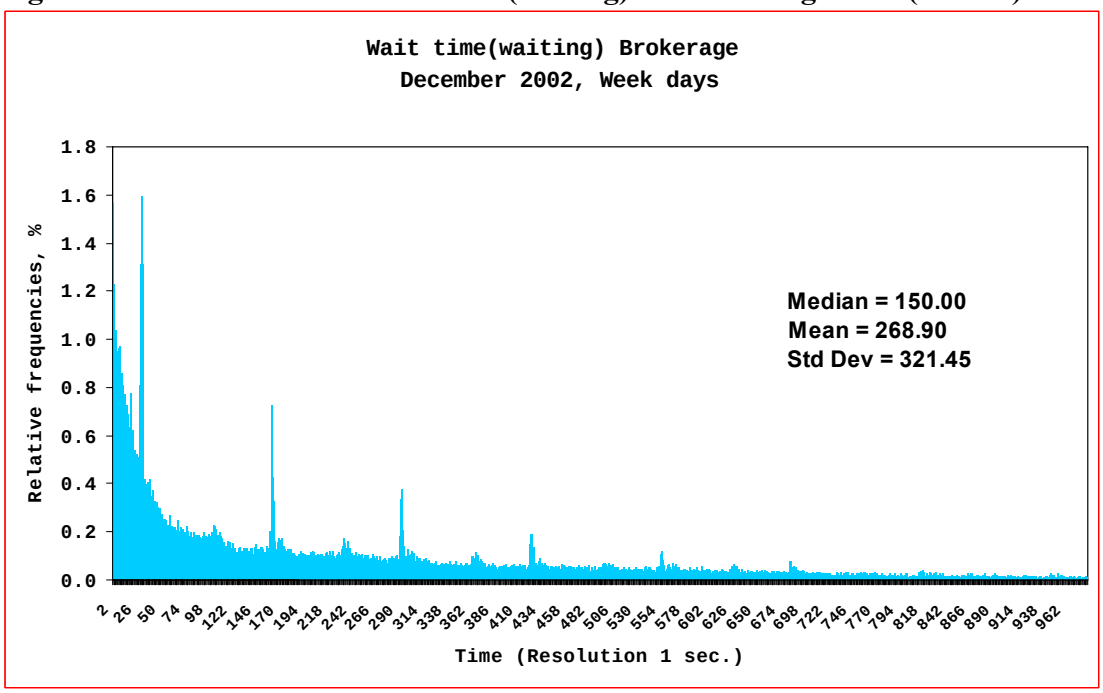


Figure 93 presents the distribution of Brokerage unhandled times. Note that this Figure is similar to Figure 92, but with modes at different times.

Figure 93 : Distribution of unhandled for Brokerage calls (Dec-02)

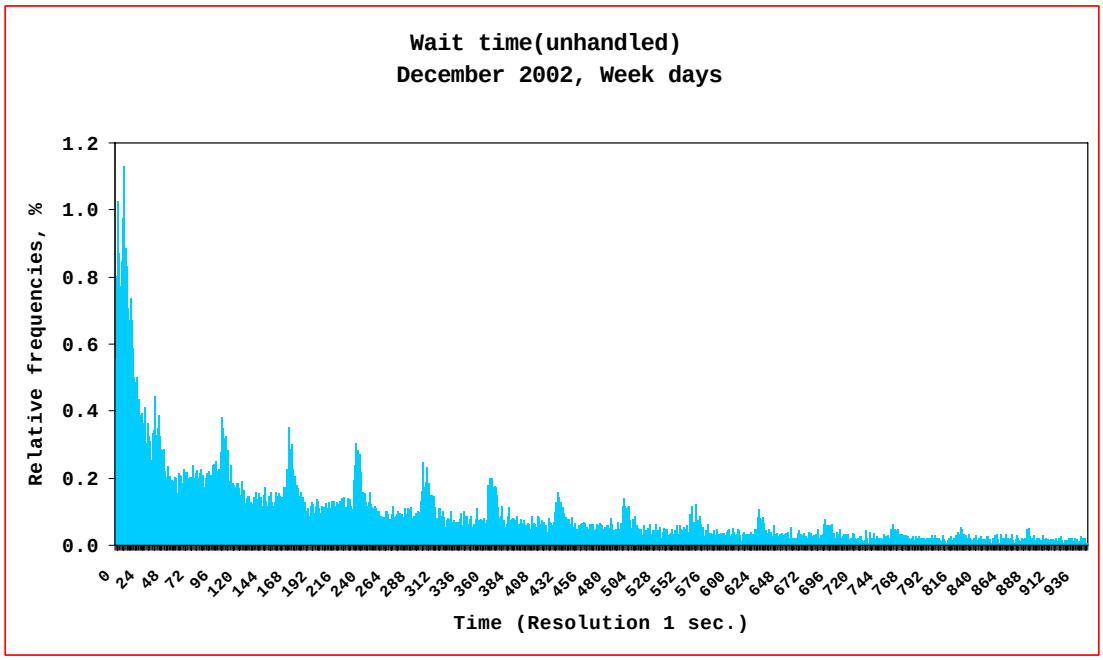


Figure 94 shows the distribution of Acquired Bank waiting times. This distribution has two modes at 14 and 23 seconds.

Figure 94 : Distribution of wait time (waiting) for Acquired Bank calls (May-01)

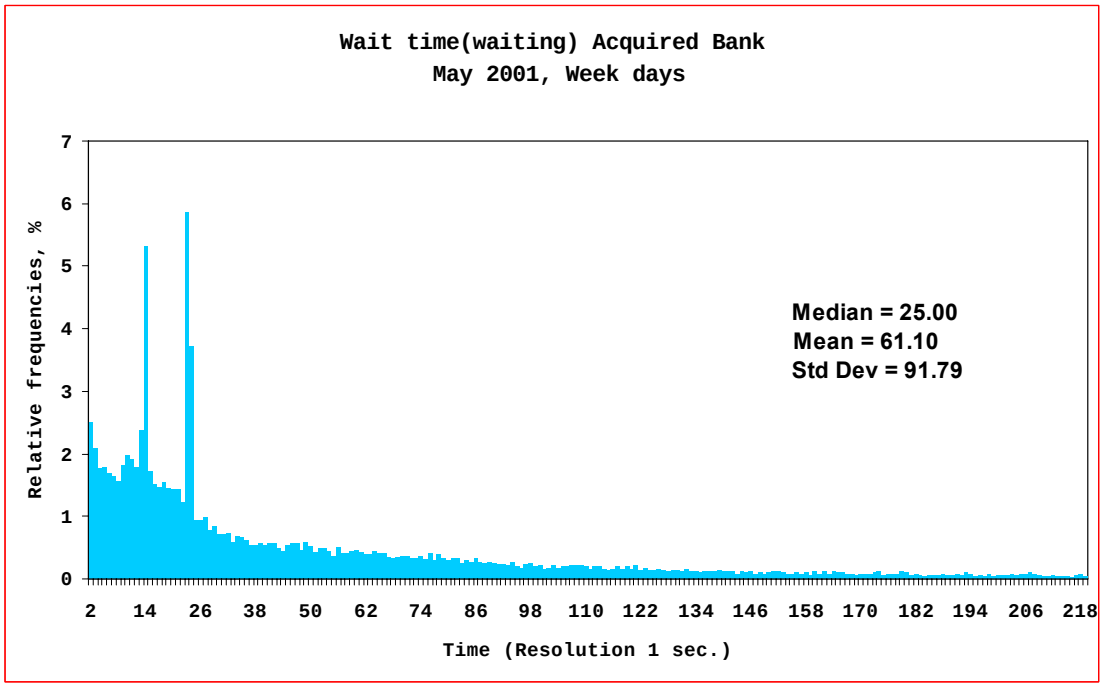


Figure 95 displays the distribution of Acquired Bank unhandled wait times. Note three modes at 3, 10 and 49 seconds.

Figure 95 : Distribution of unhandled for Acquired Bank calls (May-01)

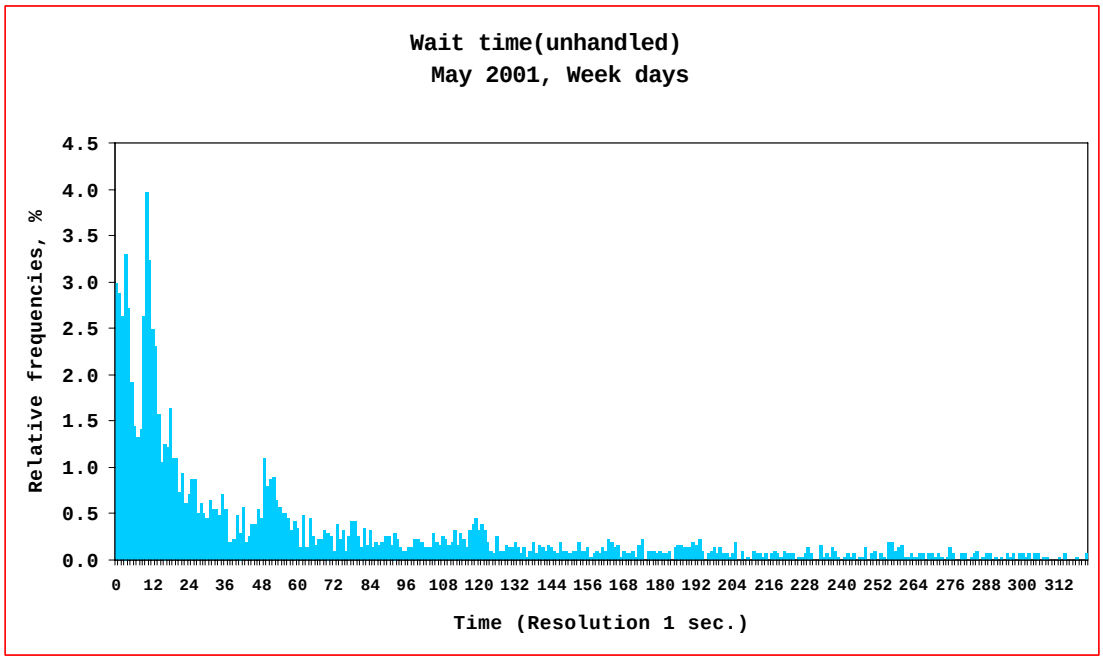


Figure 96 presents the distribution of Consumer Loans waiting times (Note that there also exist observations larger than 90 seconds). Note three modes at 6 (largest), 19 and 26 seconds. Note also that the distribution of unhandled wait times has no obvious modes, as shown on Figure 97. We observe that most customers (20%) disconnected immediately.

Figure 96 : Distribution of wait time (waiting) for Consumer Loans calls (May-01)

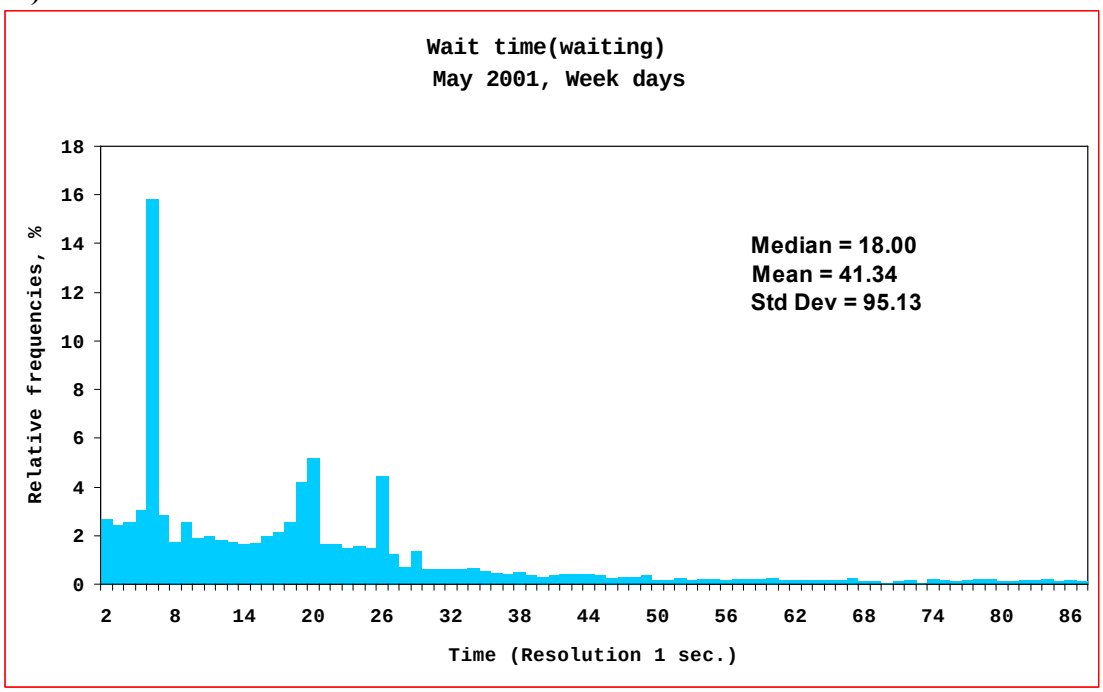
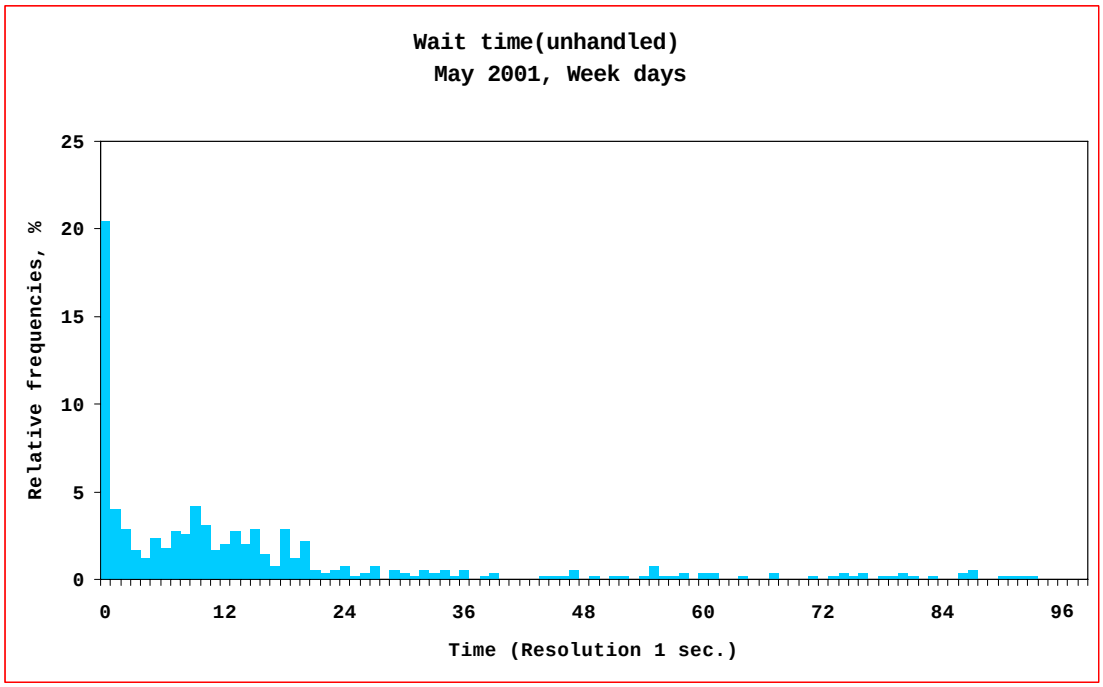


Figure 97 : Distribution of unhandled for Consumer Loans calls (May-01)



Now consider the dynamics of the waiting time distribution for different customer types during the considered period. For selected customer types we consider the waiting time distributions for three months – May 2001, May 2002 and May 2003.

Figure 98 : Dynamics of wait time (waiting) distribution for Retail calls

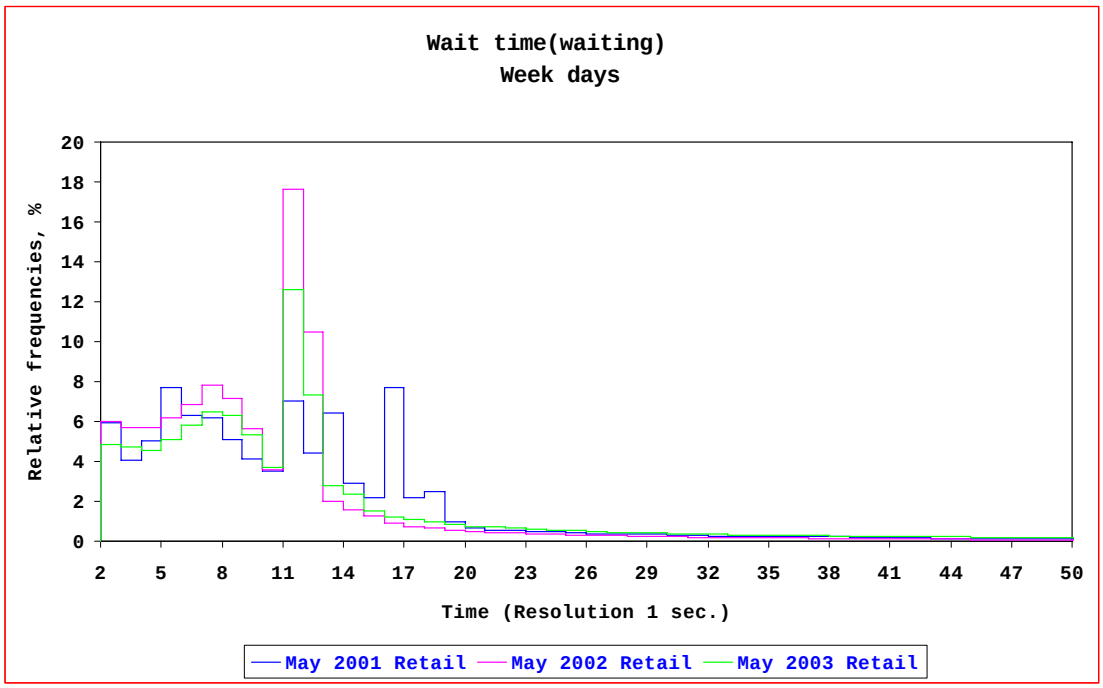


Figure 98 shows the waiting distributions for Retail calls (Note that there also exist observations larger than 50 seconds).
May 2002 and May 2003 distributions have the significant mode at 11 seconds (May-02 17.65% and May-03 12.59%). Note the mode at 16 seconds (7.68%) in May 2001.

Thus it seems plausible that the service Protocol changed between May 2001 and May 2002.

Figure 99 shows the waiting distributions for Business calls (Note that there also exist observations larger than 110 seconds). The May 2002 distribution has a mode at 5 seconds (10.59%). The May 2001 and May 2002 distributions have a mode at 19 seconds and the May 2003 distribution has a mode at 23 seconds.

Figure 99 : Dynamics of wait time (waiting) distribution for Business calls

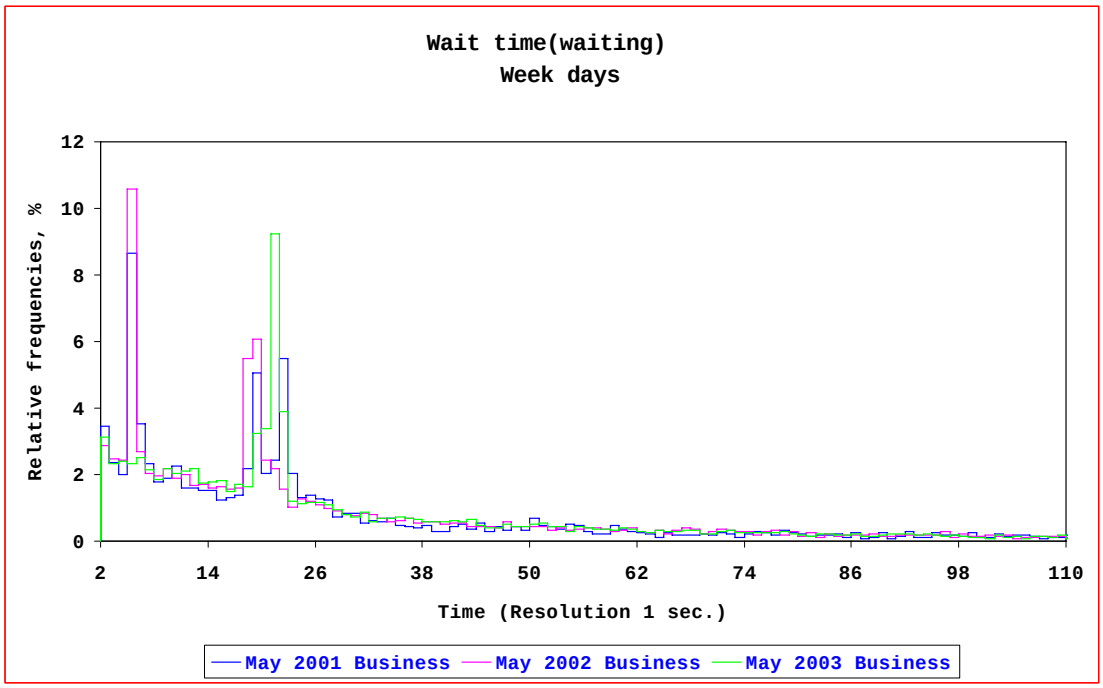


Figure 100 : Dynamics of wait time (waiting) distribution for EBO calls

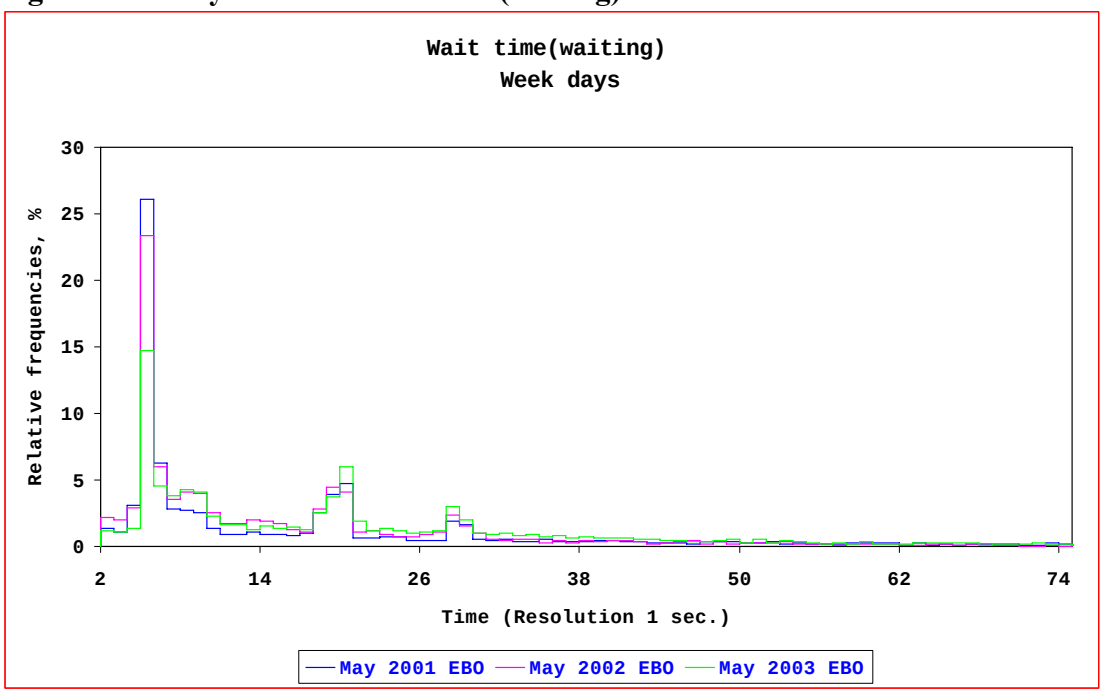


Figure 100 present the waiting distributions for EBO calls (Note that there also exist observations larger than 75 seconds). All distributions have a mode at 5 seconds.

Figure 101, Figure 102 and Figure 103 display the waiting time distributions corresponding to Telesales, Platinum and Spanish calls. Note that May 2001 distribution differs from the other distributions.

Figure 101 : Dynamics of wait time (waiting) distribution for Telesales calls

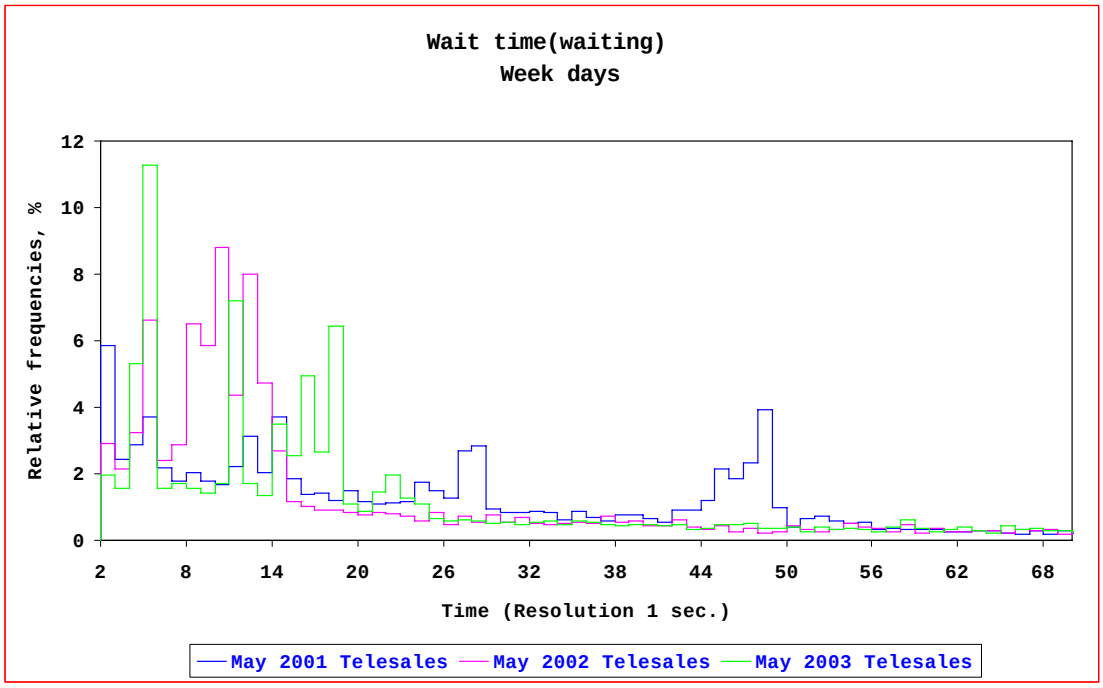


Figure 102 : Dynamics of wait time (waiting) distribution for Platinum calls

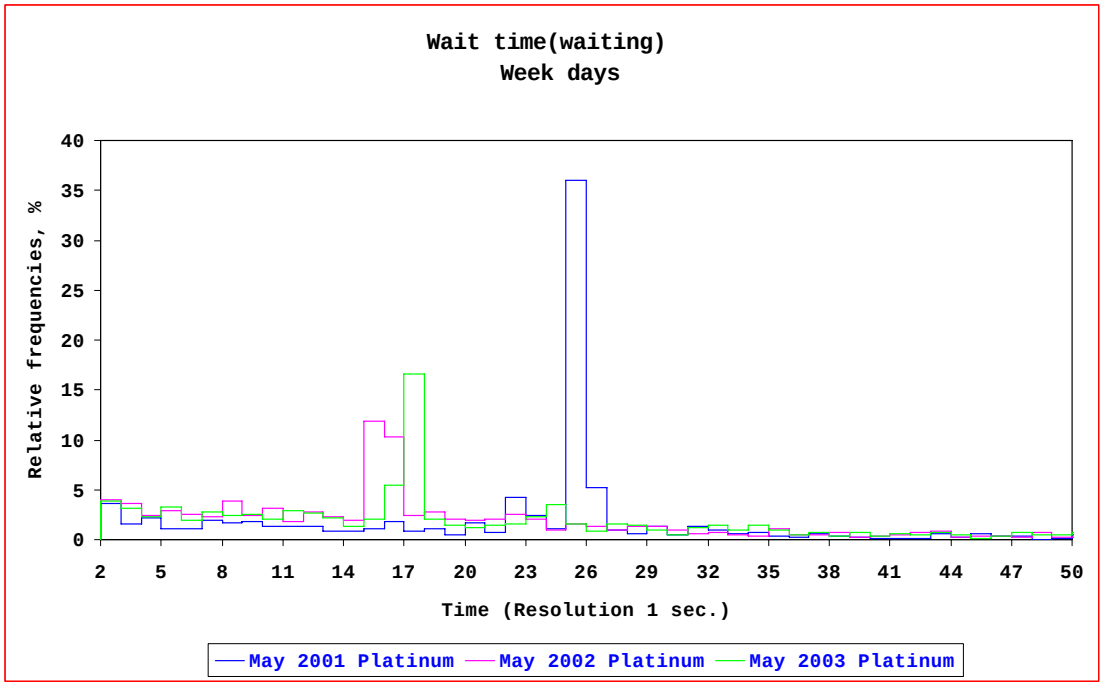
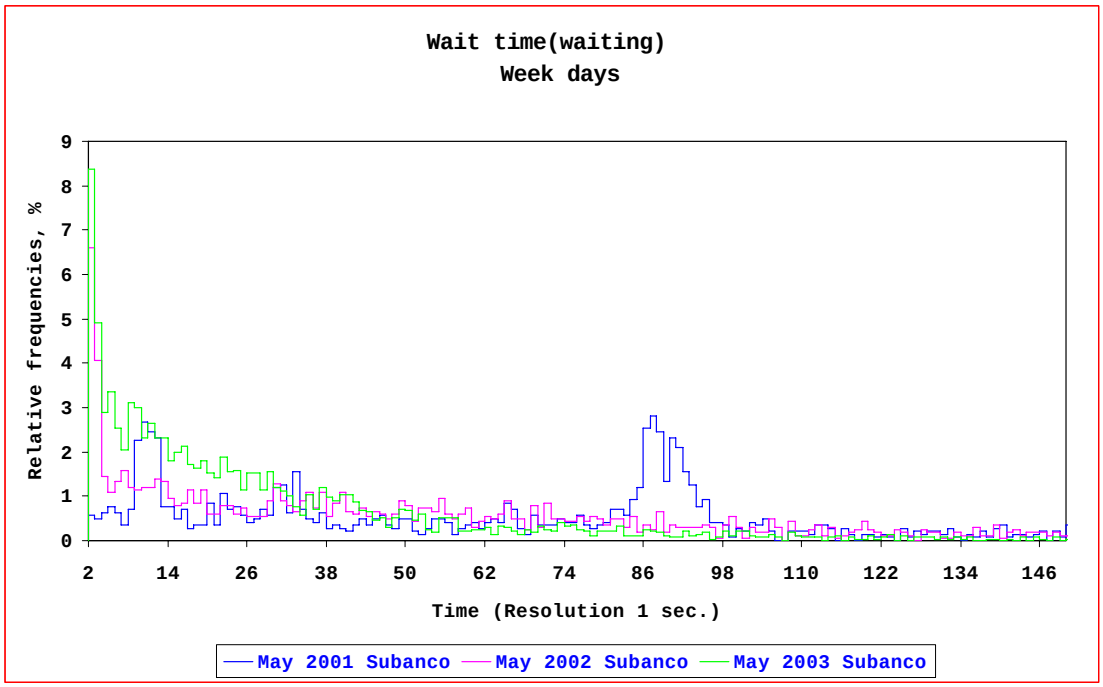


Figure 103 : Dynamics of wait time (waiting) distribution for Spanish calls



We now consider hourly dynamics of average wait time (all) for Retail and Telesales customer types. Figure 104 shows the dynamics of average wait time for Retail calls. Note that dynamics is not regular, the peaks occur at different times. We see three peaks took place during periods 12pm-2am, 6am-8am and 6pm-8pm in May 2001. We observe a unique peak between 6am and 8am in May 2002 and 2003.

Figure 104 : Dynamics of average wait time (all) for Retail calls (May-01, May-02, May-03)

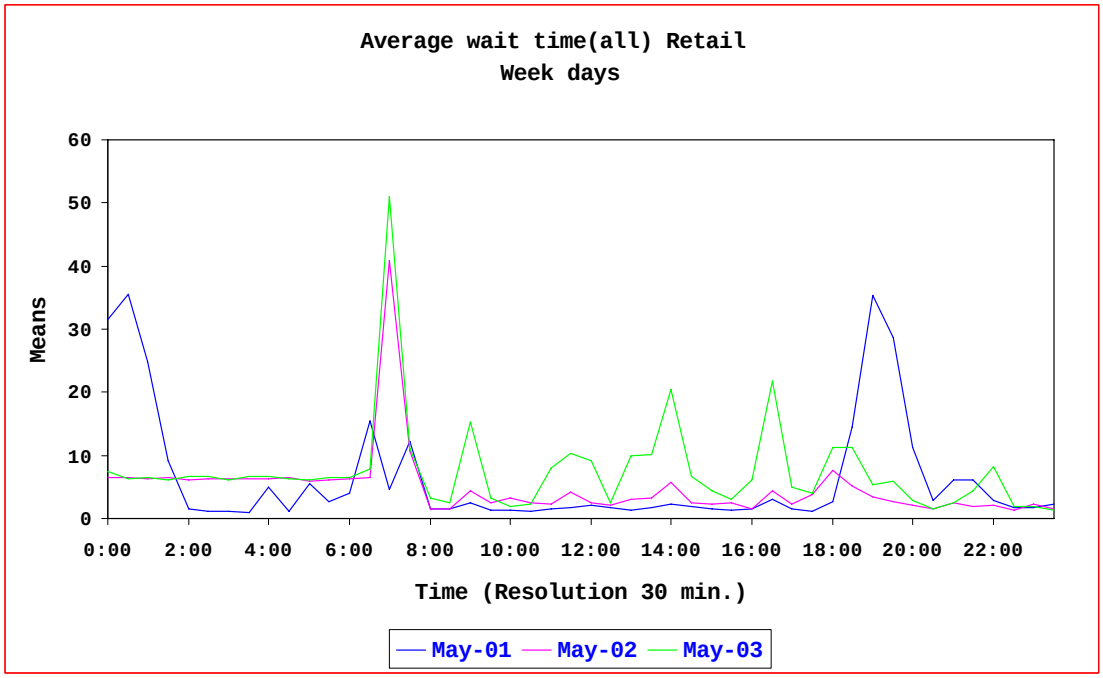
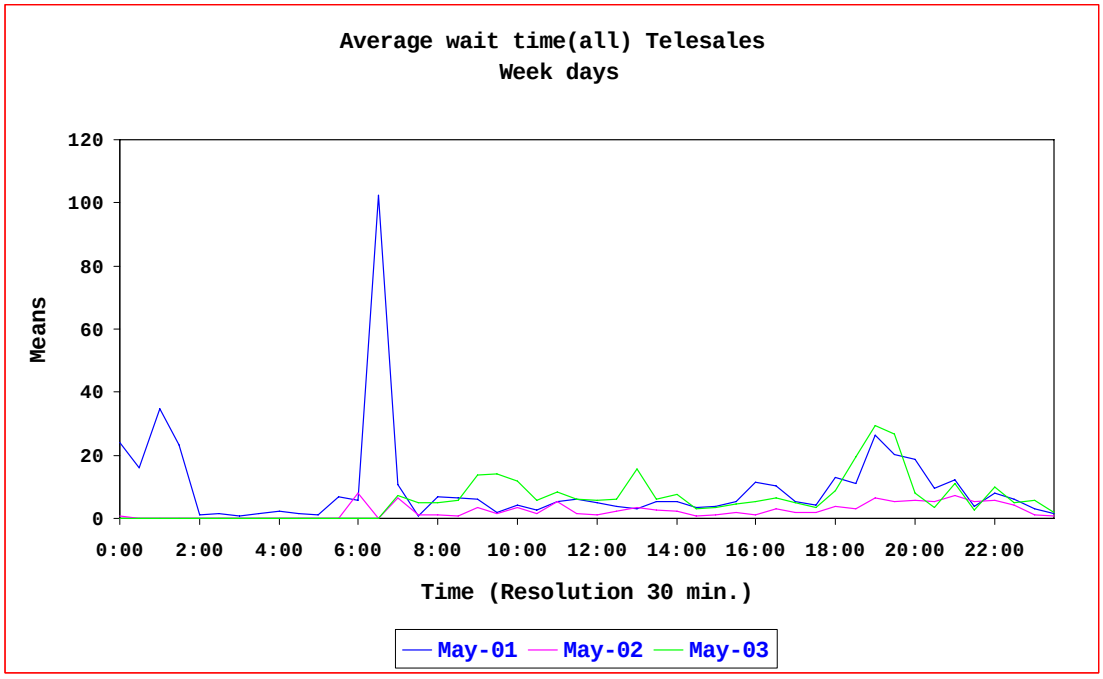


Figure 105 presents the dynamics of average wait time for Telesales calls. We see three peaks took place in periods 12pm-2am, 6am-8am and 6-8pm on May 2001. We have no significant peak in May 2002 and we observe a unique peak between 6pm and 8 pm in May 2003.

Figure 105 : Dynamics of average wait time (all) for Telesales (May-01, May-02, May-03)



7.2 Service without waiting

In this subsection we consider percentage of customers who received service immediately (customers who waited 0 or 1 second).

We shall study these characteristics for different service types. Time period from 7am till 12pm will be considered.

Figure 106 presents total rate of customers who receive service immediately. We observe that percentage is relatively high (between 60 and 85%). Note the significant decrease took place from April 2002 till September 2002 (from 80% to 60%). It can be explained by minimal number of agents in this period (see Figure 123 in Subsection 8.1).

Figure 106 : Rate of Total calls served without waiting

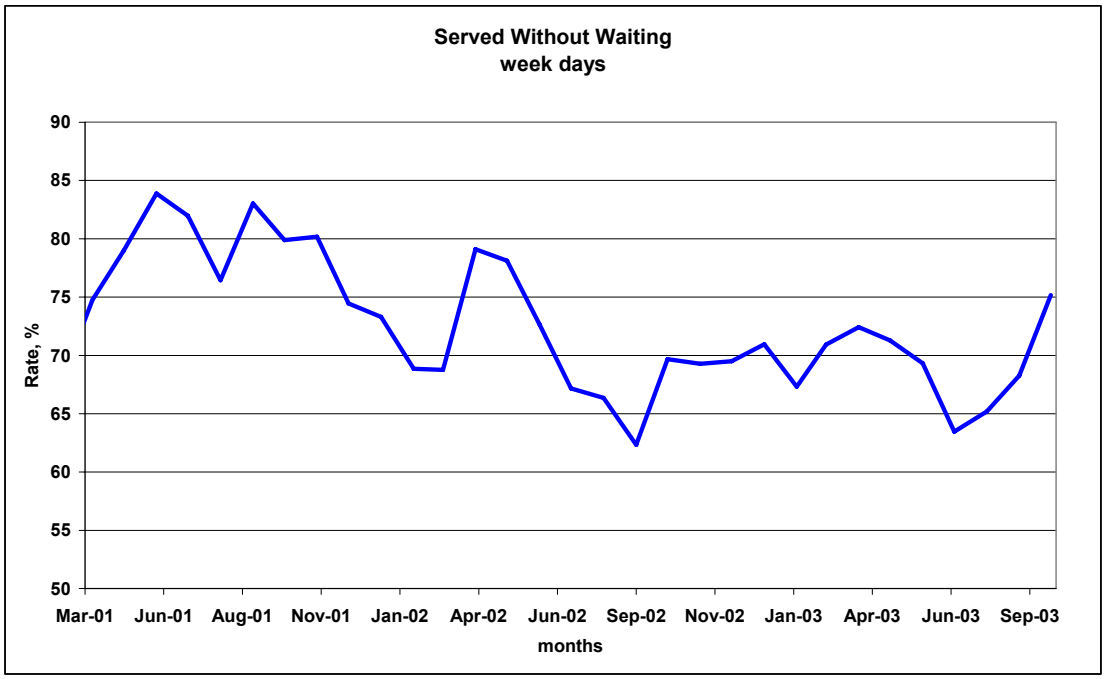


Figure 107 shows the rate of immediately served Retail and Premier calls during all period. We observe that the rate is less for Premier calls for all months, although Premier are VIP Retail customers. However, average waiting time for Retail calls is larger for most months than the one for Premier calls (see Figure 108).

Figure 107 : Rate of Retail and Premier calls served without waiting

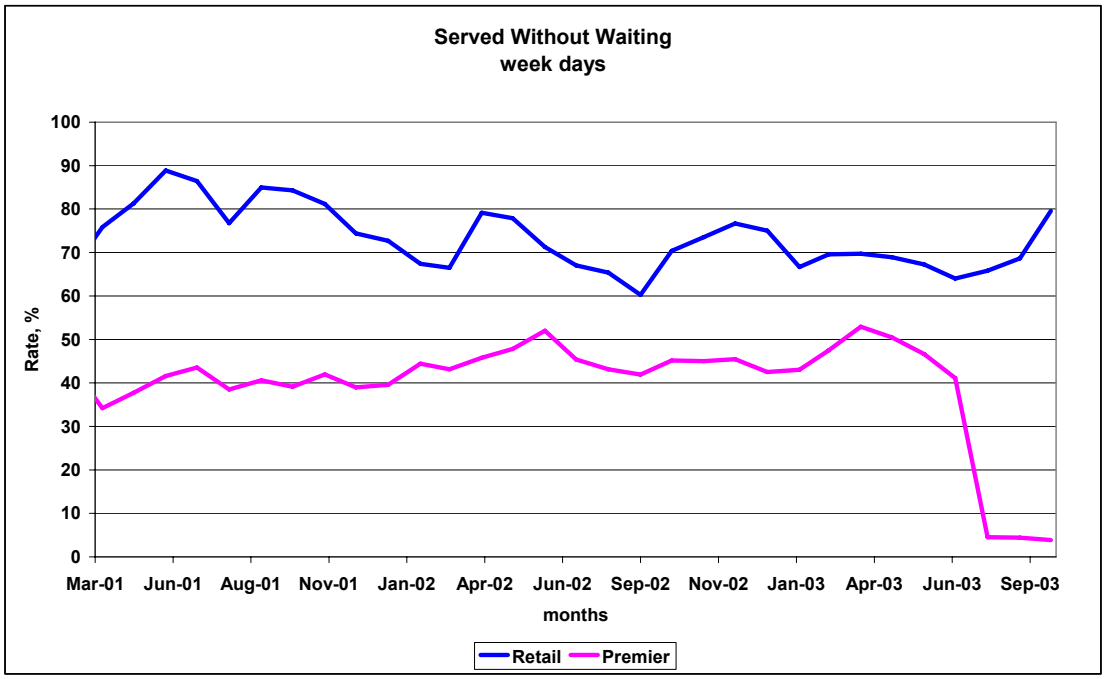


Figure 108 : Average wait time (all) for Retail and Premier calls

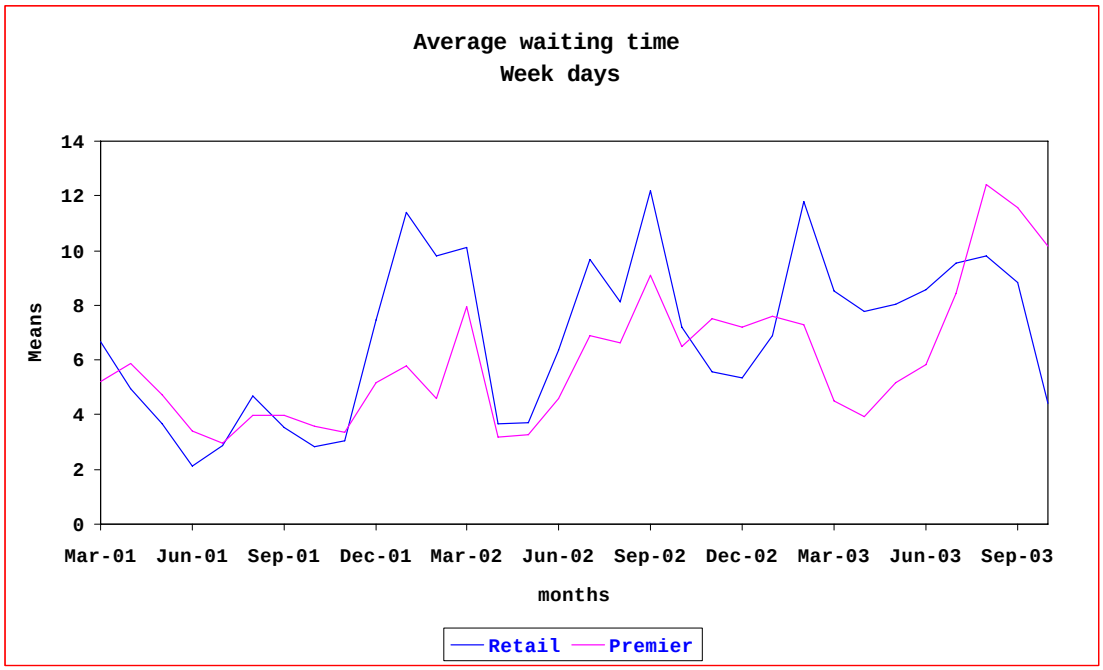


Figure 109 presents Retail and Premier rates of customers served without waiting in August 2002. Note that this rate is smaller for Premier calls, although Premier is high priority of Retail. Note also that the average waiting time for Retail calls is larger than the one for Premier calls (see Figure 110). The reason is that 30% of Premier calls receive service at 3 seconds (see Figure 111). We consider this phenomenon below specifically for August 2002.

Figure 109 : Rate of Retail and Premier calls served without waiting (Aug-02)

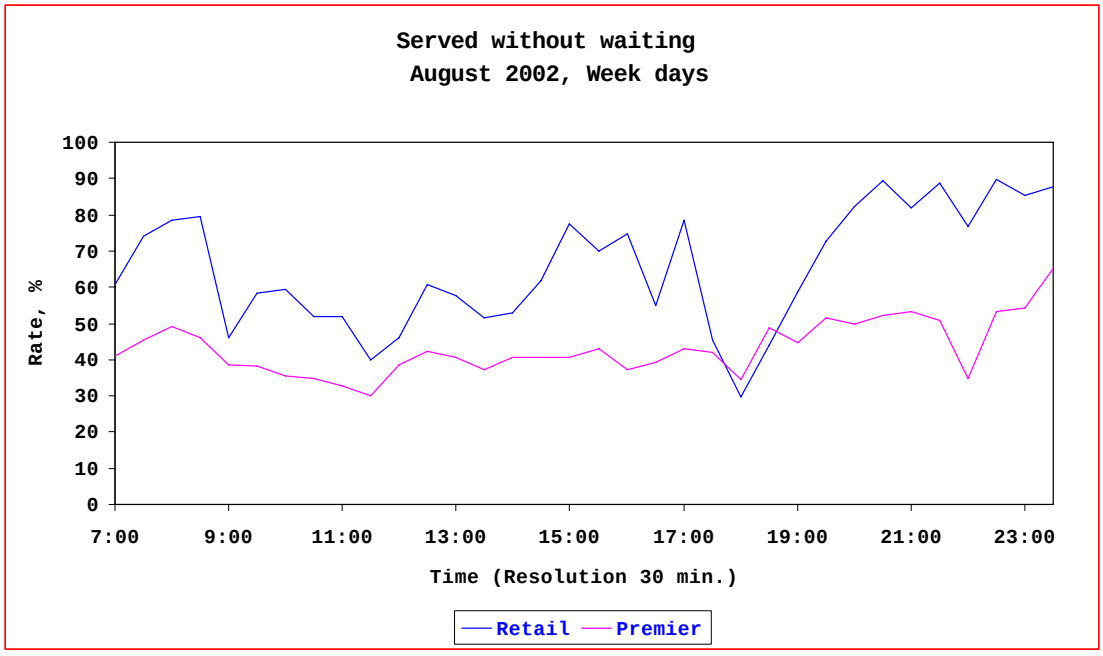


Figure 110 : Average wait time (all) for Retail and Premier calls (Aug-02)

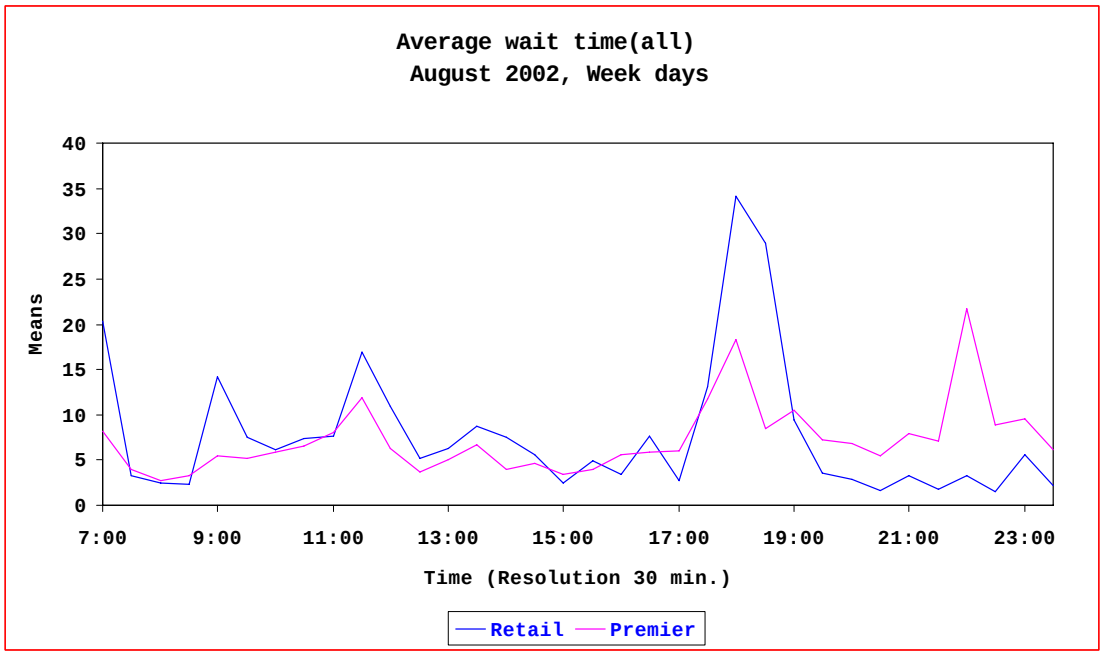


Figure 111 : Distributions of wait time (all) for Retail and Premier calls (Aug-02)

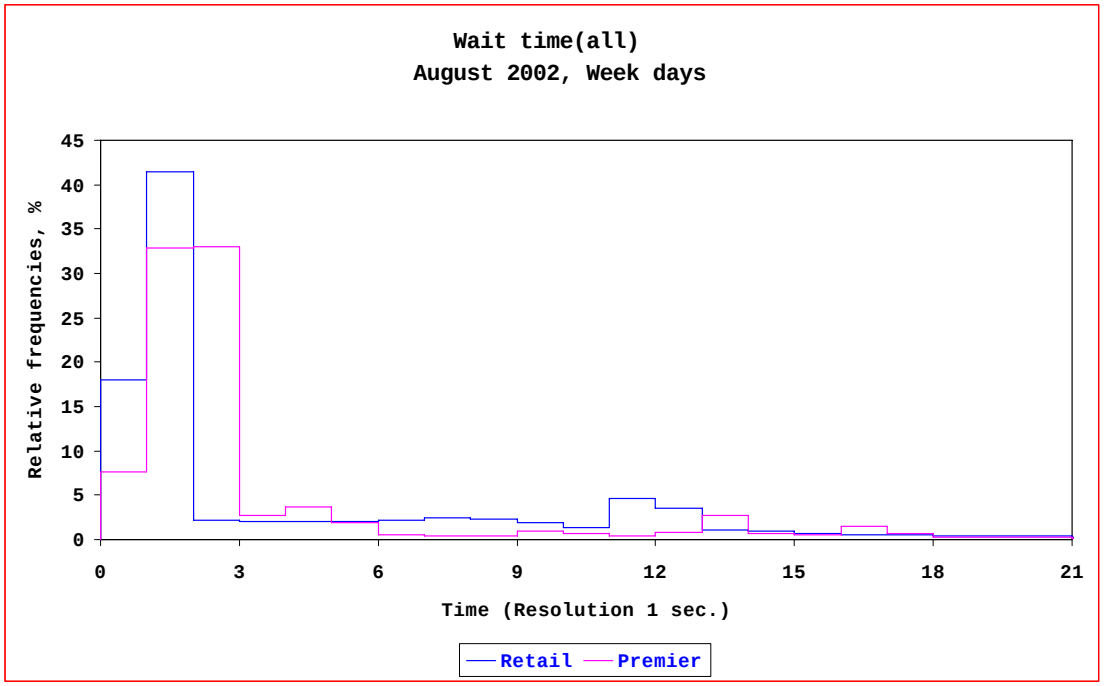


Figure 112 presents the rate of immediately served Business and Platinum calls during all period. We observe that the rate is less for Business calls for all months, except November 2002. Recall that Platinum is the high priority version of Business customers. Note also that the average waiting time for Business calls is larger than the one for Platinum calls from March 2002.

Figure 112 : Rate of Business and Platinum calls served without waiting

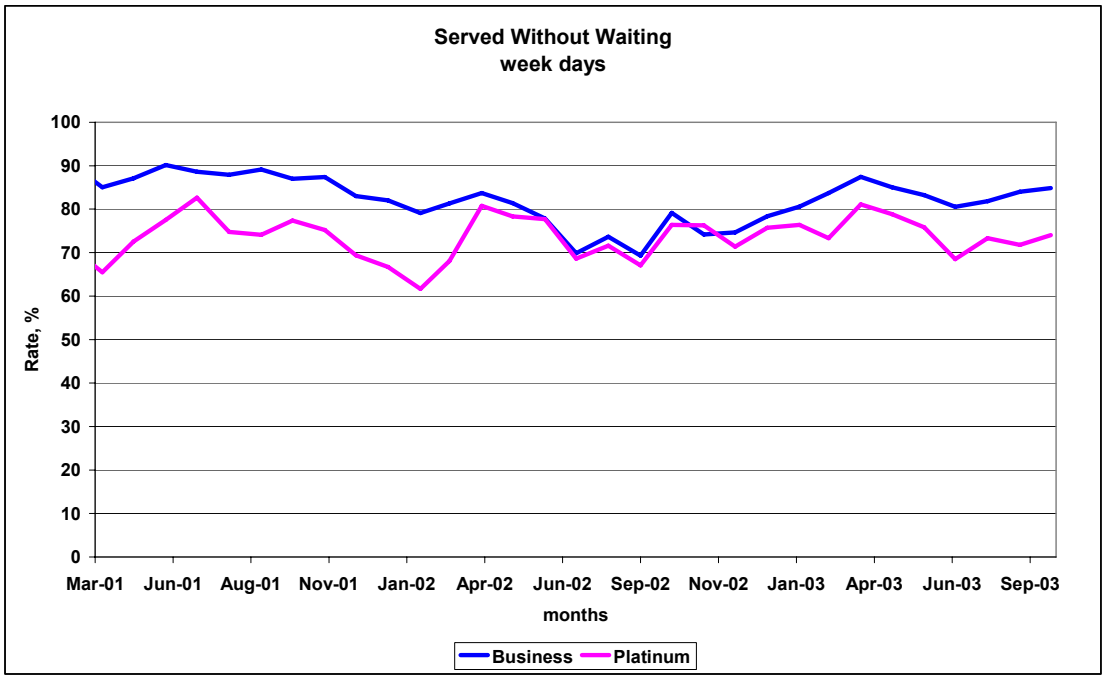


Figure 113 : Average wait time (all) for Business and Platinum calls

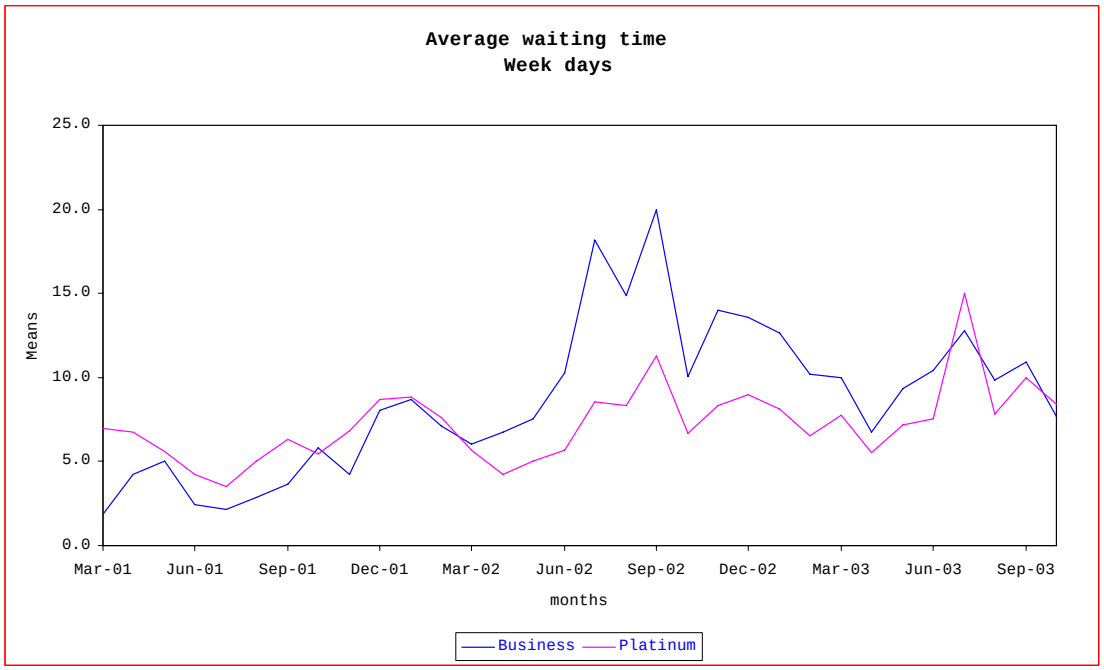


Figure 114 presents Business and Platinum rates of customers served without waiting in February 2002. We observe that the rate is smaller for Platinum calls, although Platinum is high priority of Business. Note also that the average waiting time for Platinum calls is larger than the one for Platinum calls except the period between 4:30pm and 6:30pm (see Figure 115).

Figure 114 : Rate of Business and Platinum calls served without waiting (Feb-01)

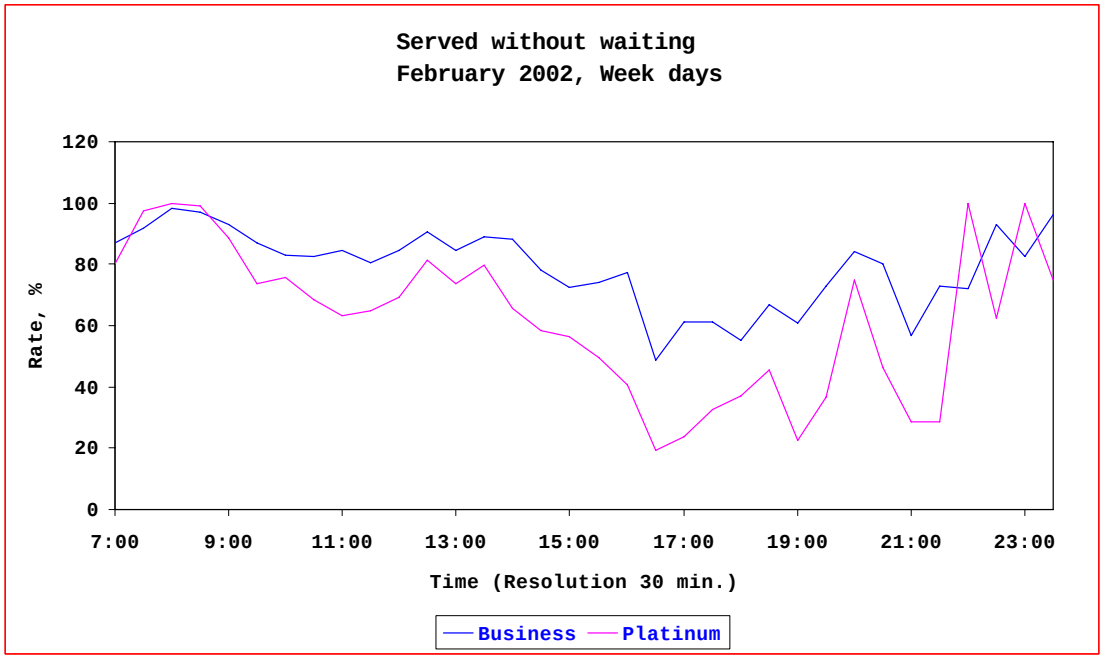
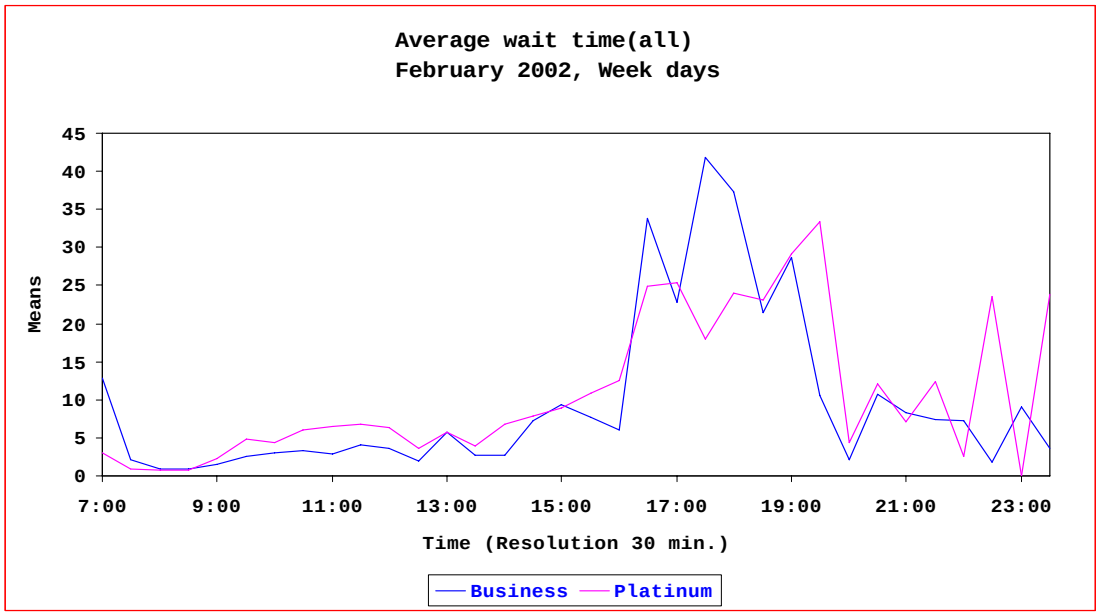


Figure 115 : Average wait time (all) for Business and Platinum calls (Feb-01)



We now consider hourly dynamics of the rate of immediately served customers for different types of calls.

Figure 116 presents dynamics of Telesales rate of calls served without waiting in September, 2001, September, 2002 and September, 2003. Note that it lowest in September 2002. We observe also that September 2001 and September 2003 rates are similar until 2:30pm; then September 2003 rate is larger until 4:30pm and after 4:30pm September 2001 rate is larger. Dynamic of average waiting time corresponds to scenario described above. As shown in Figure 117, average waiting time is largest

in September 2002. It is larger in September 2001 between 2:30pm and 5pm, and then it is larger in September 2003.

Figure 116 : Dynamics of rate served without waiting for Telesales calls (Sept-01, Sept-02 and Sept-03)

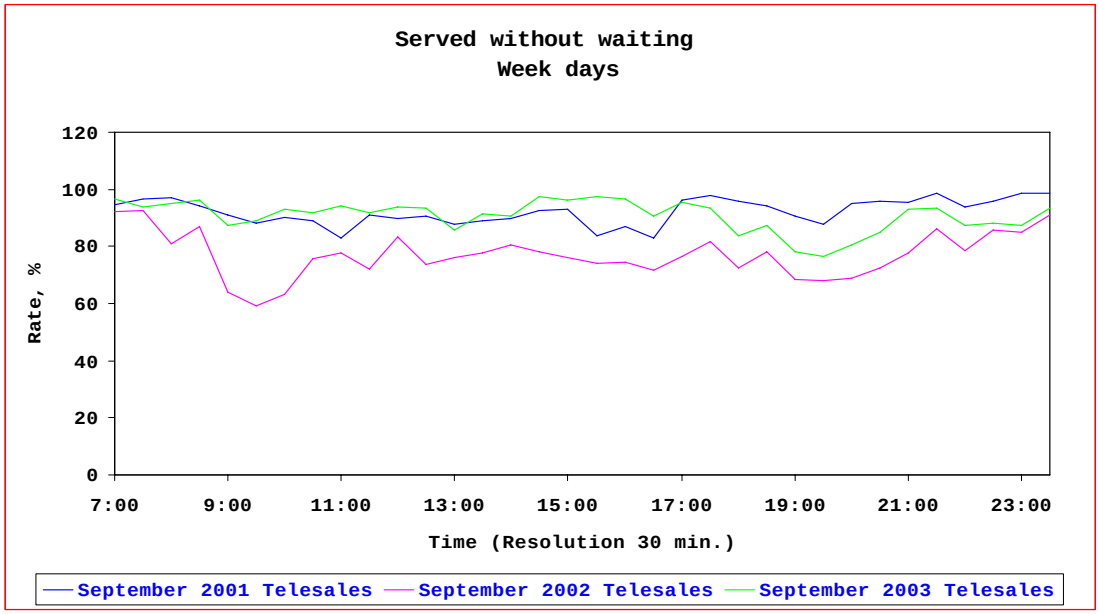


Figure 117 : Dynamics of average wait time (all) for Telesales calls (Sept-01, Sept-02 and Sept-03)

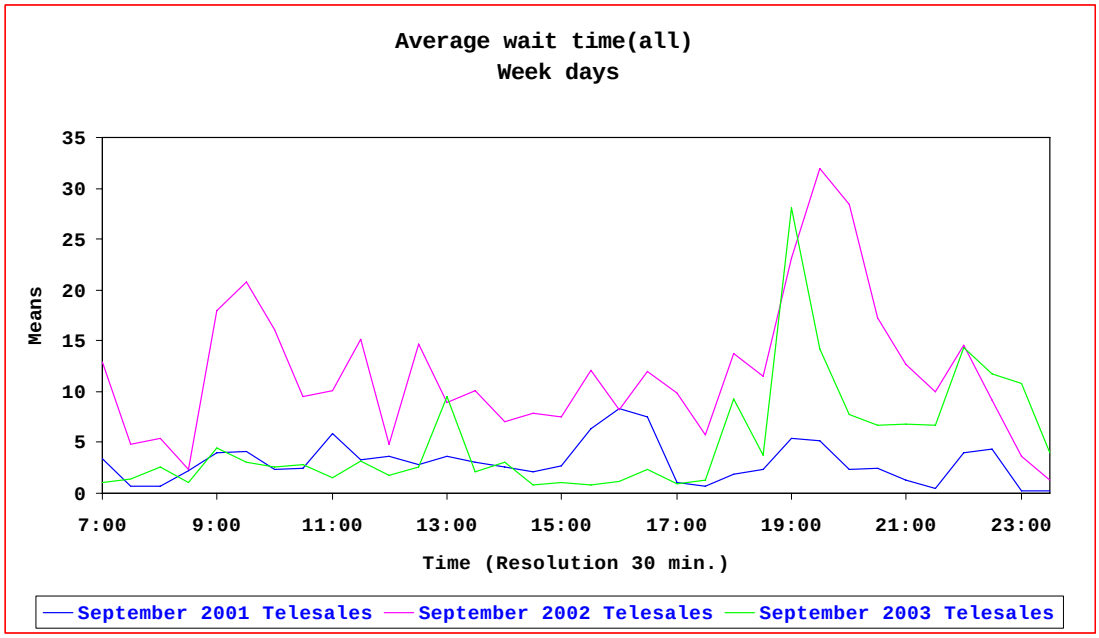


Figure 118 displays dynamics of Spanish rate of calls served without waiting in September, 2001, September, 2002 and September, 2003. We observe that 100% of calls that arrived between 5:30pm and 9:30pm received service immediately. Note, however, that waiting time is positive (for 30% of calls) in this time period. There is the following explanation of this phenomenon. All Spanish calls are switched to English node after 5:30pm and, then, they received service without waiting. Thus,

customers first waited a positive time in Spanish node and, then, receive service immediately in English node.

Figure 118 : Dynamics of rate served without waiting for Spanish calls (Sept-01, Sept-02 and Sept-03

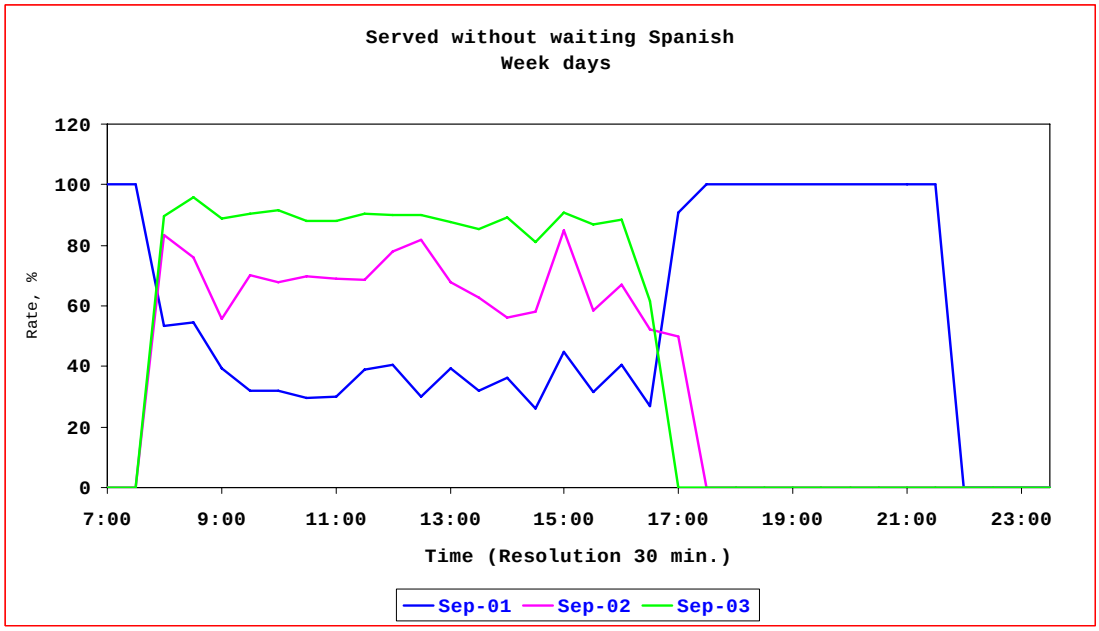
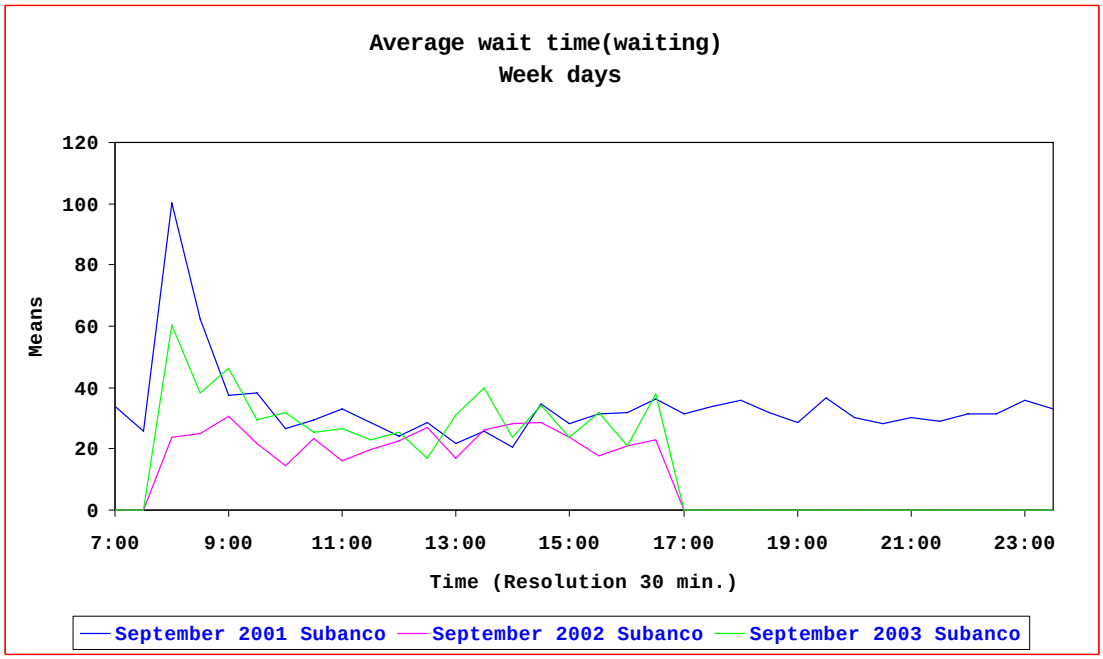


Figure 119 : Dynamics of average wait time (all) for Spanish calls (Sept-01, Sept-02 and Sept-03)



7.3 Abandonment

There are the following types of unhandled calls in the system:

- Abandons – customers that abandon after 5 or more seconds of wait.
- Short abandons – customers that abandon after 4 or less seconds of wait.

- Other unhandled

In this subsection we study overall unhandled calls and their dynamics for different customer types.

Figure 120 displays fraction of unhandled calls per weekday. Note that fraction is small (about 1-3 %) and it reaches a peak (about 5%) in December 2002. A peak can be explained by large unhandled fraction of Priority Service (28%), Brokerage (21%) and Mortgage (25%) calls in December 2002 (see Table 17 below).

Figure 120 : Unhandled rate per weekday

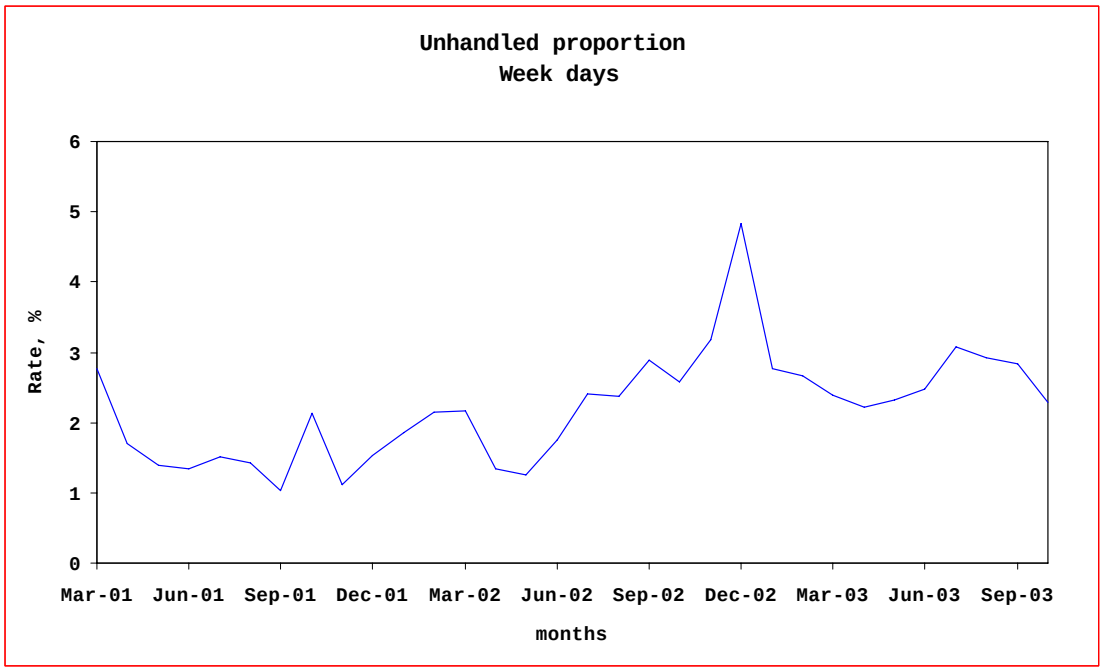


Figure 121 presents the dynamics of unhandled calls of Retail type. Note the peak in May 2001 of unhandled calls occurs between 6pm and 8pm.

Figure 122 presents the dynamics of unhandled calls among Telesales calls. Note that the fraction of unhandled calls is very small in May 2002, which means a high quality of service. In May 2003, unhandled calls have local peaks at 9am, 1pm and 7pm. In May 2001 unhandled calls have a single significant peak around 7pm.

Figure 121 : Dynamics of unhandled Retail calls (May-01, May-02, May-03)

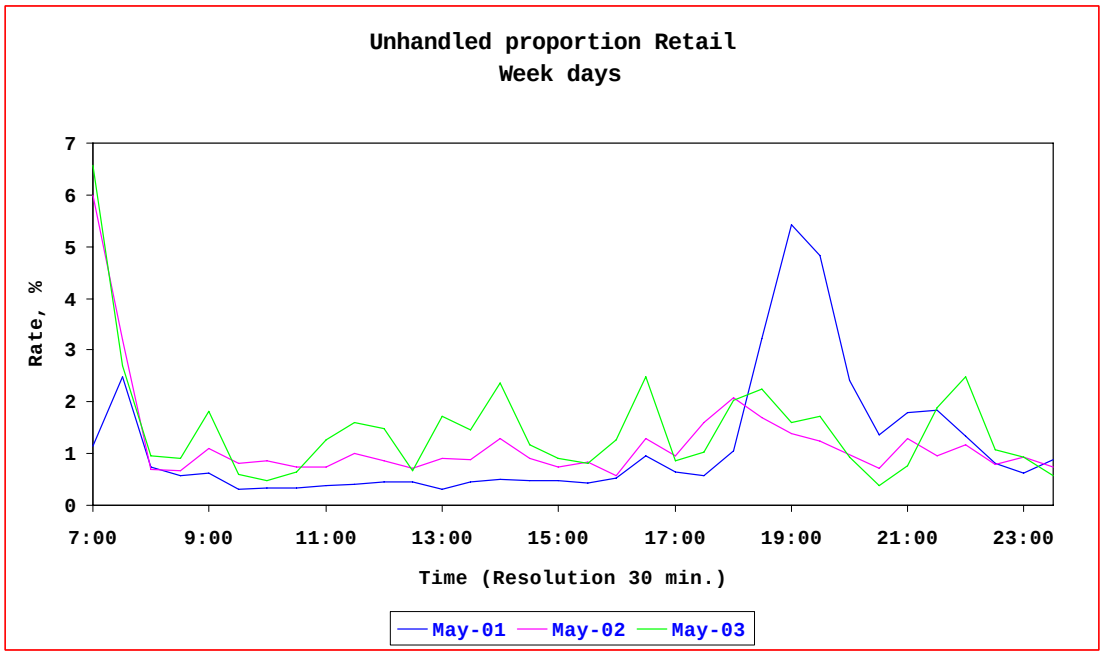


Figure 122 : Dynamics of unhandled Telesales calls (May-01, May-02, May-03)

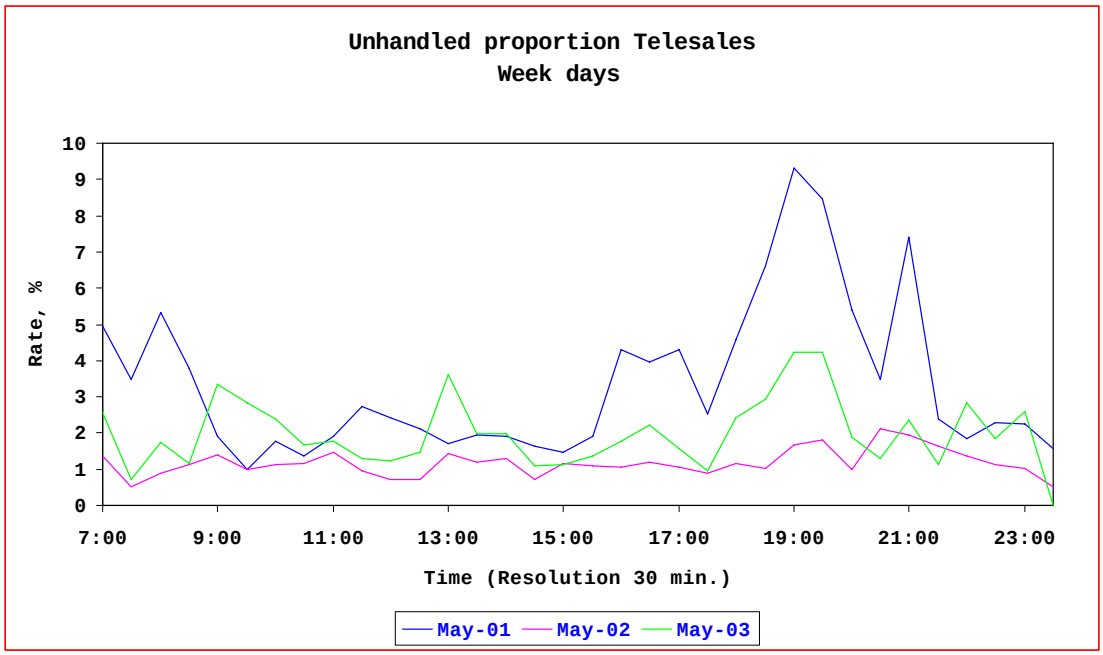


Table 19: Unhandled proportion per month, 2001

Date	Mar-01	Apr-01	May-01	Jun-01	Jul-01	Aug-01	Sep-01	Oct-01	Nov-01	Dec-01
Retail	1.63	1.15	0.87	0.61	0.66	1.05	0.81	0.73	0.86	1.34
Premier	2.55	1.89	1.69	1.47	1.52	1.68	1.76	1.43	1.46	1.92
Business	1.84	1.74	1.66	1.19	1.35	1.45	1.39	1.64	1.54	2.34
Platinum	1.98	1.64	1.60	1.26	1.06	1.49	1.31	1.50	1.44	2.03
Consumer Loans	1.10	1.09	0.96	0.68	1.62	2.23	2.10	4.57	2.61	1.96
Online Banking	0.55	0.39	0.34	0.25	0.75	0.61	0.56	0.73	0.52	0.97
EBO	5.95	3.06	2.19	1.22	1.69	2.61	1.80	1.05	1.22	2.16
Telesales	12.69	6.37	2.95	4.05	6.14	2.77	1.41	12.41	1.40	1.55
Spanish	5.28	7.87	11.49	11.23	24.39	31.54	7.40	7.98	8.19	4.64
CCO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	14.22	0.86	5.08
Acquired Bank	3.33	1.57	3.04	4.14	6.19	0.00	0.00	0.00	0.00	0.00

Table 20 : Unhandled proportion per month, 2002

Date	Jan 02	Feb 02	Mar 02	Apr 02	May 02	Jun 02	Jul 02	Aug 02	Sep 02	Oct 02	Nov 02	Dec 02
Retail	1.82	2.15	2.23	1.27	1.07	1.63	1.99	1.91	2.27	1.78	1.43	1.04
Premier	2.05	1.96	2.73	1.63	1.70	2.81	3.21	3.11	3.26	2.89	2.64	2.74
Business	2.52	2.64	2.58	2.48	2.76	3.23	4.89	4.20	5.38	3.37	3.88	3.77
Platinum	2.79	2.73	2.78	2.20	2.62	2.31	3.66	3.49	4.33	2.76	3.11	3.98
Consumer Loan	1.82	2.13	1.37	1.16	1.32	1.73	2.28	2.00	1.89	1.56	2.21	1.49
Online Banking	1.31	1.59	2.19	0.82	1.13	1.28	2.64	1.97	2.44	2.26	1.92	2.22
EBO	1.92	1.83	1.50	0.99	1.27	2.08	1.63	1.89	3.23	2.70	2.97	2.77
Telesales	1.40	2.00	1.54	1.27	1.19	1.12	3.10	2.38	2.83	3.16	2.67	1.14
Spanish	1.65	1.83	3.19	2.91	2.04	2.62	1.63	1.77	1.69	1.27	0.93	1.38
Case Quality	0.00	0.00	0.00	0.00	0.00	0.00	3.74	11.15	19.76	15.41	16.34	12.52
Priority Service	0.00	0.00	0.00	0.00	10.07	10.90	24.81	34.10	34.70	31.70	29.31	28.59
AST	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.77	2.26
CCO	3.45	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.86	9.50
Brokerage	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.01	20.94
Mortgage	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	29.20	25.60
BPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51	2.50

Table 21: Unhandled proportion per month, 2003

Type	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03
Retail	1.19	1.72	1.44	1.37	1.38	1.47	1.63	1.58	1.51	0.87
Premier	2.66	2.80	2.27	2.40	2.71	2.73	3.55	5.04	4.90	4.95
Business	3.23	3.00	2.73	2.06	2.48	2.85	3.17	3.03	3.83	2.48
Platinum	3.39	3.47	3.25	2.57	2.83	3.07	3.19	3.23	4.27	3.67
Consumer Loan	2.44	2.15	0.95	0.70	1.01	1.40	1.28	1.35	0.88	0.86
Online Banking	1.74	1.56	1.72	0.86	0.84	0.94	3.03	2.70	3.06	1.85
EBO	2.89	3.71	2.31	3.02	2.92	2.23	1.92	1.56	2.27	1.36
Telesales	1.89	2.42	1.78	1.52	2.08	2.30	4.25	3.17	1.10	1.39
Spanish	1.87	2.83	1.79	2.37	2.21	1.72	1.84	1.54	1.39	0.88
Case Quality	12.79	13.32	12.79	11.87	11.77	11.36	15.16	17.53	17.40	16.09
Priority Service	29.50	30.12	26.75	25.10	24.58	24.69	21.83	24.60	25.09	22.10
AST	0.86	0.81	0.39	0.56	1.24	1.82	2.13	2.91	1.63	1.17
CCO	3.58	3.25	3.79	3.67	4.21	3.77	5.05	4.87	5.03	4.57
Brokerage	8.59	3.13	2.87	2.76	2.42	2.91	4.45	2.72	3.03	1.75
Mortgage	20.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BPS	2.14	3.65	5.64	4.14	3.99	6.96	8.49	5.56	12.18	18.69

Main observations:

- High service level (< 3% unhandled calls) is observed for most service types.
- On the other hand, the unhandled fraction for Premier calls is larger than the one for Retail calls in most months; and the unhandled fraction for Platinum calls is larger than the one for Business calls in most months. Recall that Premier is the high priority version of Retail and Platinum is the high priority version of Business.
- Bad service quality levels are observed for some customer types, such as Priority Service and Mortgage.
- Outlier cases are observed. Note, example in October 2001 for Telesales calls.

8 Number of agents

8.1 Average number of agents per weekday

In this subsection we consider the average number of agents per month from March 2001 until October 2003. First, note that the "number of agent" variable in the "Time series" option provides us with the overall number of agents who were logged on for a specific day.

We expect the number of agents to be related to the arrival rate of incoming calls ("arrivals to offered"). An increase (decrease) in the arrival rate could be followed by an increase (decrease) in the number of agents. If the number of agents is not "tuned" to the arrival rate, we can expect dramatic changes in performance measures, such as average waiting time and fraction abandoning.

Figure 123 shows the average number of agents per weekday from March 2001 till October 2003. We see that the average number of agents decreases in the period from July 2001 to September 2002, which could be related to the decrease in number of arrivals occurring in that period (see Figure 58). After September 2002 the number of arrivals increases and average number of agents increases too and reaches a peak (1199) in March 2003.

Figure 123 : Average number of agents per weekday for Total calls

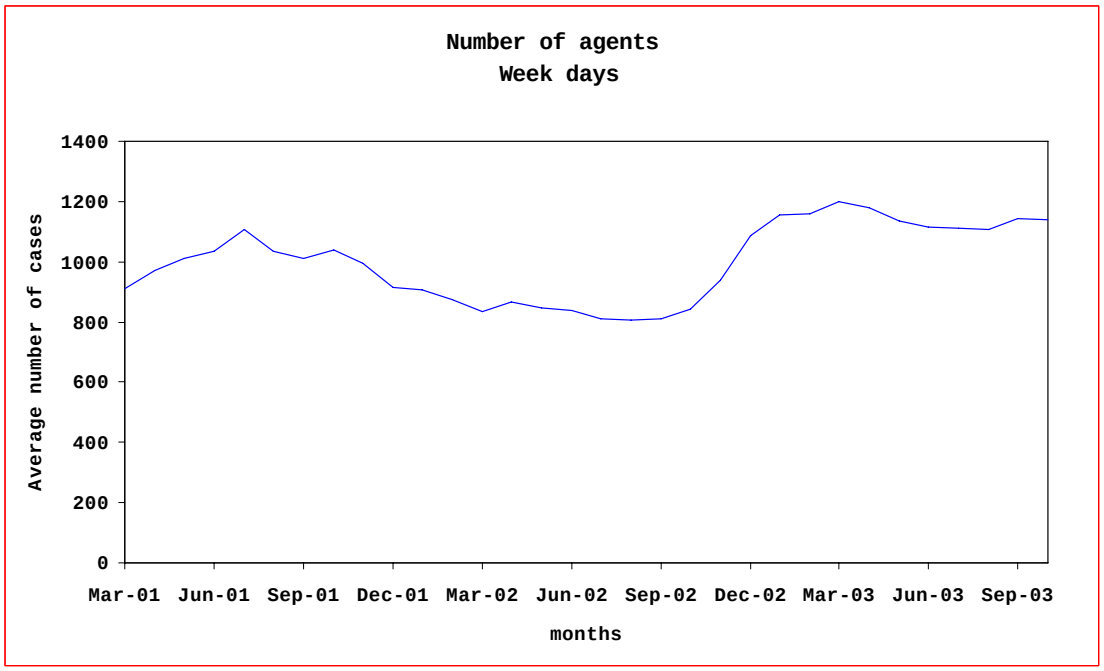


Figure 124 : Average number of agents per weekday for Retail calls

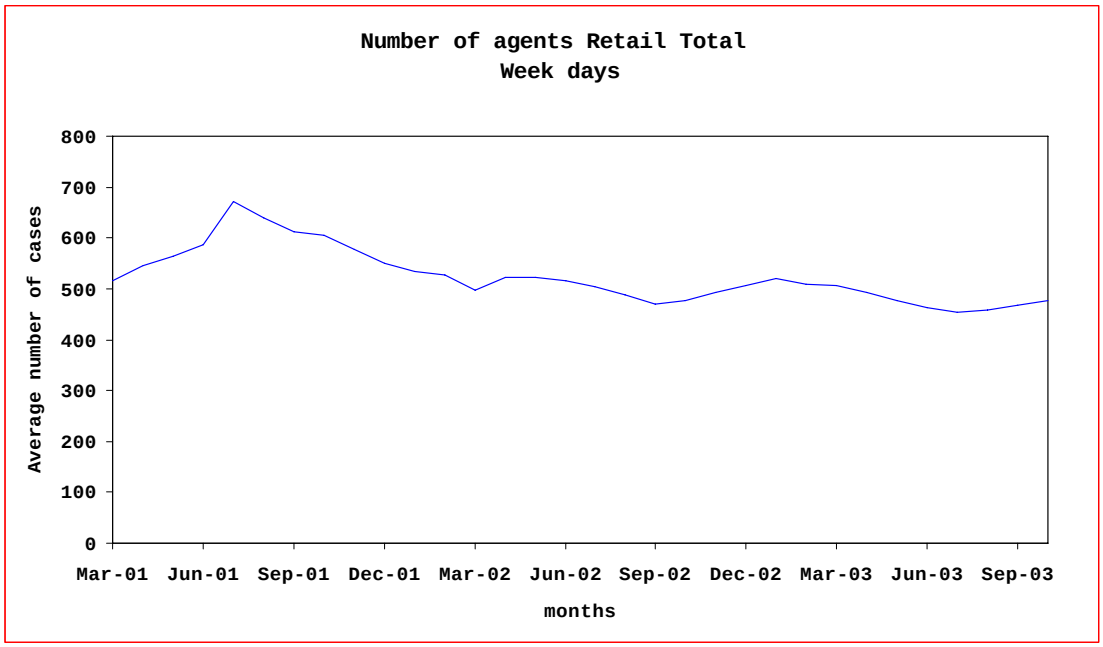


Figure 124 shows the average number of agents per weekday for Retail calls. Note that the average number of agents reaches a peak in July 2001, and then it decreases. The pattern of "arrivals to offered" for Retail calls has a similar structure: peak in July 2001 and then a decrease in arrival rate (see Figure 58). This phenomenon can be related to the one we observed in Figure 36: the proportion of Retail customers who use VRU only increases from July 2001.

Figure 125 presents the average number of agents per weekday for Consumer Loans calls. Note a significant decrease in the average number of agents in the period from November 2001 until July 2002. We observe also decrease of arrival volume in the period from October 2001 till May 2001 (see Figure 71).

Figure 125 : Average number of agents per weekday for Consumer Loans calls

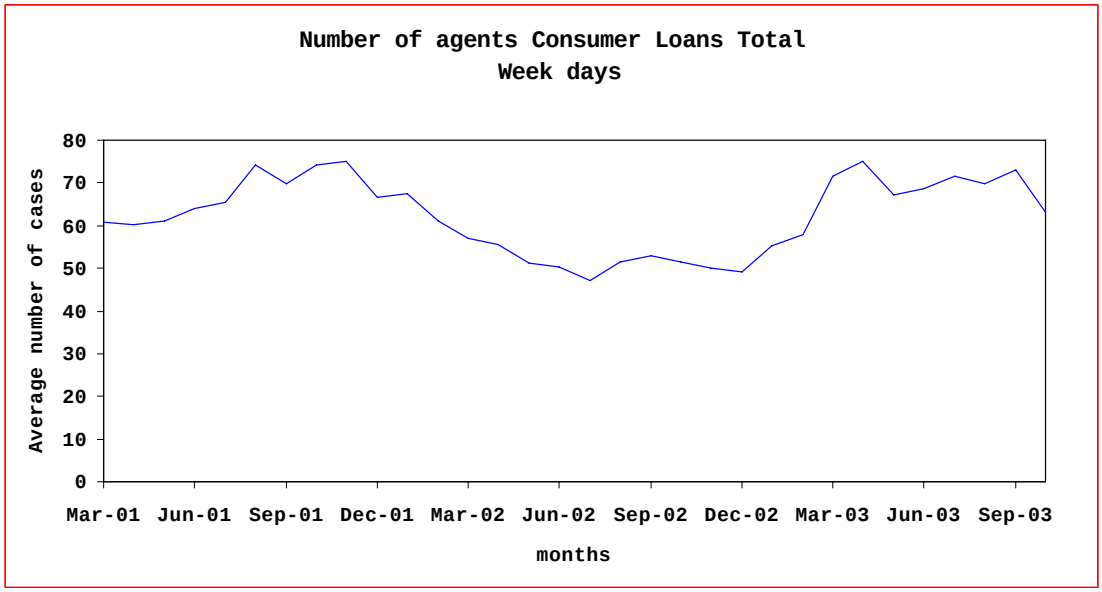


Figure 126 shows the average number of agents per weekday for Business calls. Note that the average number of agents decreases from October 2001 until July 2002 and then it remains stable. As shown in Figure 61, the reduction in arrival rate is not so steep as the reduction in number of agents from January 2002. In addition, we observe a significant increase of average wait during the period from March 2002 until July 2002 (see Figure 88). One explanation might be the increase in average customer service time during this period (see Figure 127).

Figure 126 : Average number of agents per weekday for Business calls

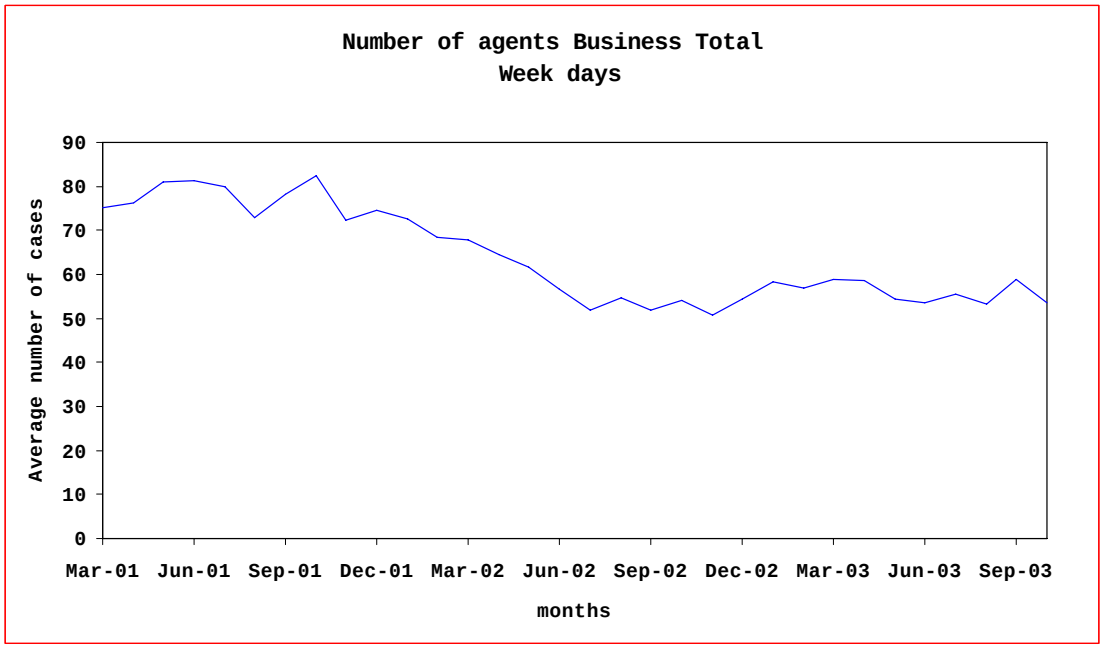


Figure 127 : Average customer service time for Business calls

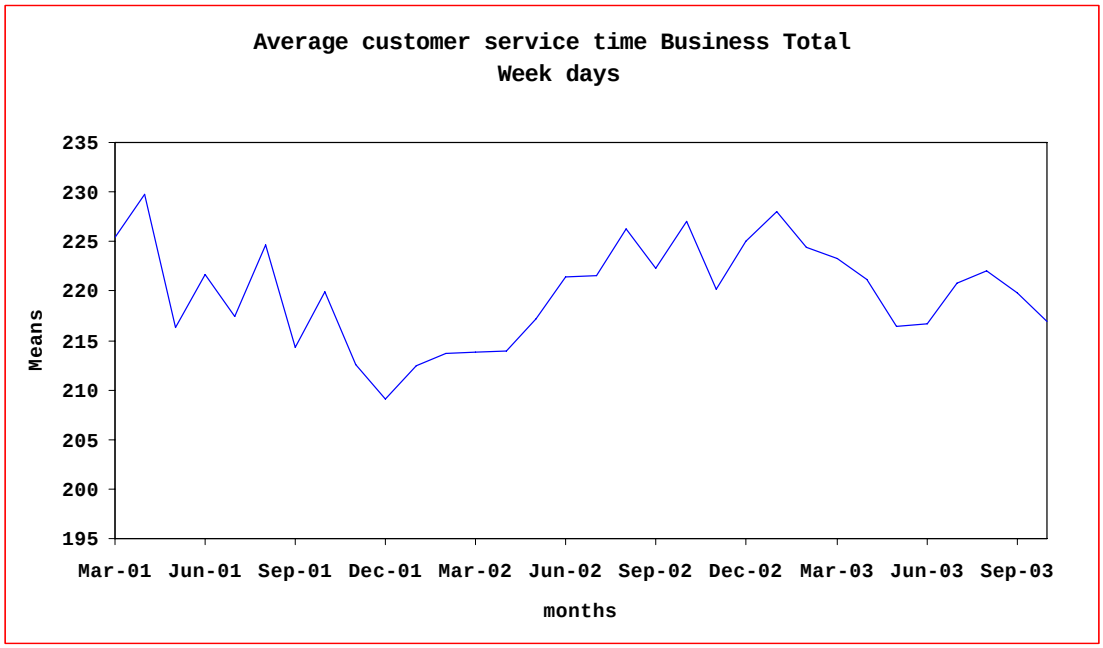


Figure 128 displays the average number of agents per weekday for Telesales calls. Note that the average number of agents remains stable, except for the period from September 2001 to February 2002. Note also that the average number of agents

reaches its maximum on October 2001 and in this month, in particular, on October, 10 we observe a significant number of arrivals (see Figure 7).

Figure 128 : Average number of agents per weekday for Telesales calls

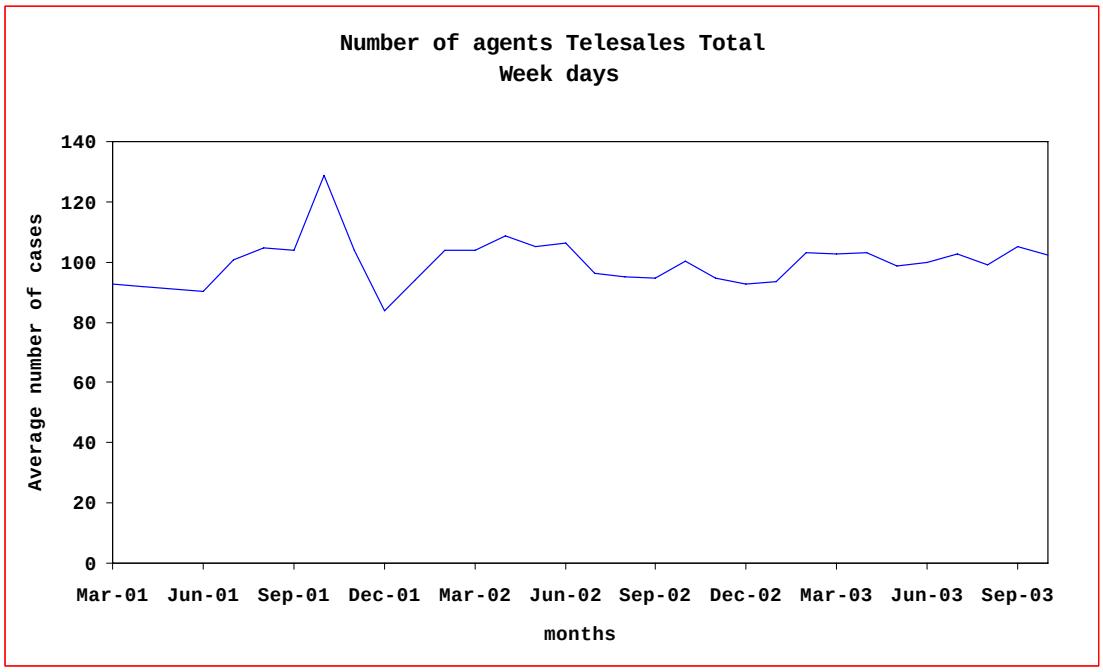


Figure 129 shows the average number of agents per weekday for Case Quality calls. Note the sharp increase in the average number of agents in the period from July 2002 till February 2003. As shown in Figure 70 the number of Case Quality arrivals increased from July 2002 until November 2002.

Figure 129 : Average number of agents per weekday for Case Quality calls

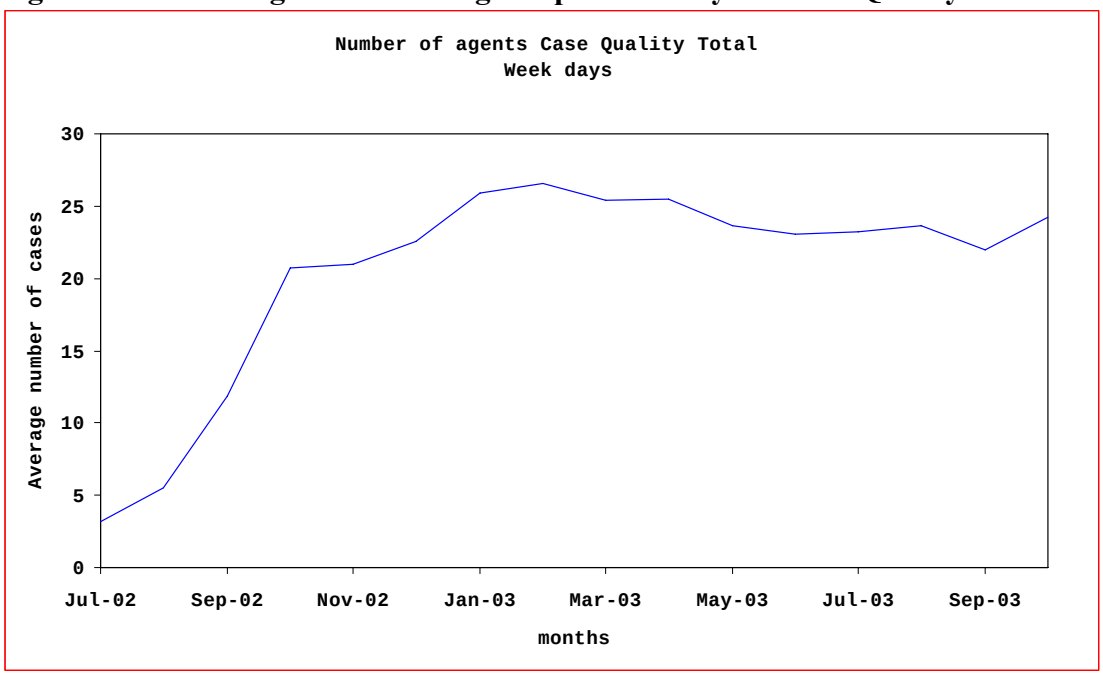


Figure 130 presents average number of agents per weekday for Spanish calls. Note the relatively small number of agents. As shown in Figure 63, the number of arrivals

increases with a sharp rise in July-August 2001. It probably explains the peak of average waiting time that occurred in August 2001 (see Figure 131). In addition, the average customer service time increases significantly from July 2001 till August 2001 (see Figure 132).

Figure 130 : Average number of agents per weekday for Spanish calls

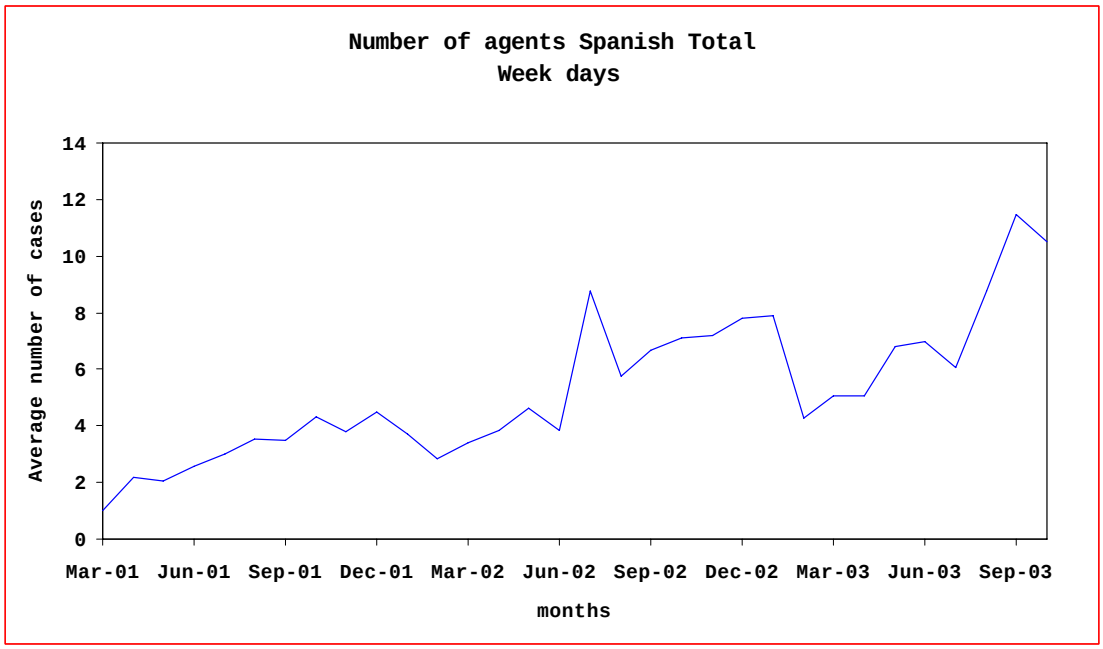


Figure 131 : Average waiting time for Spanish calls

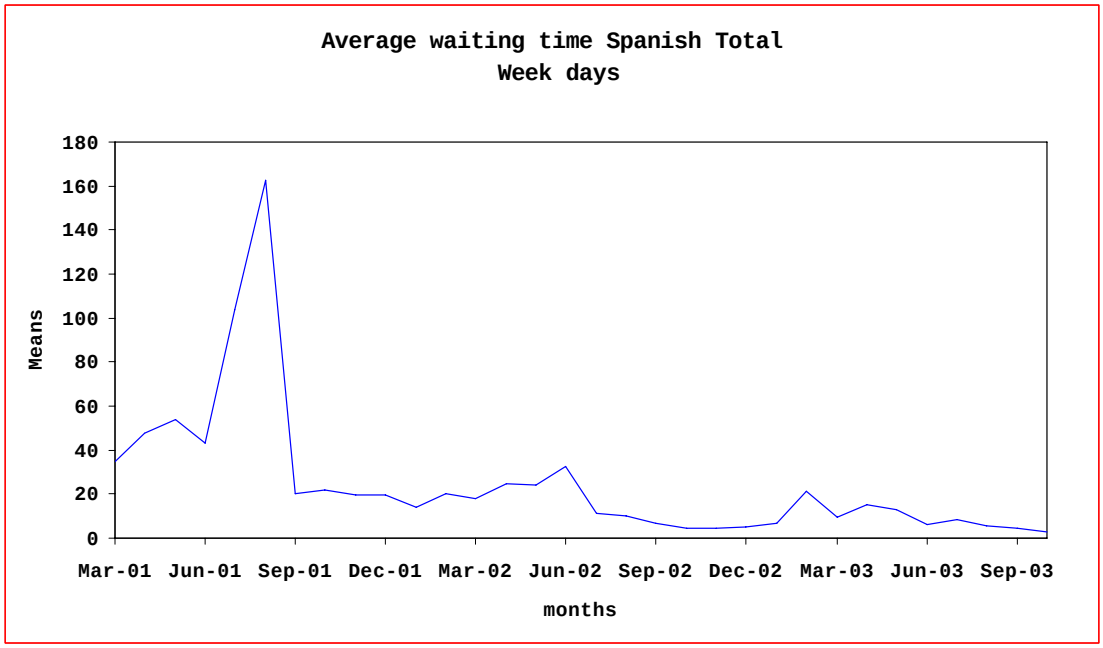
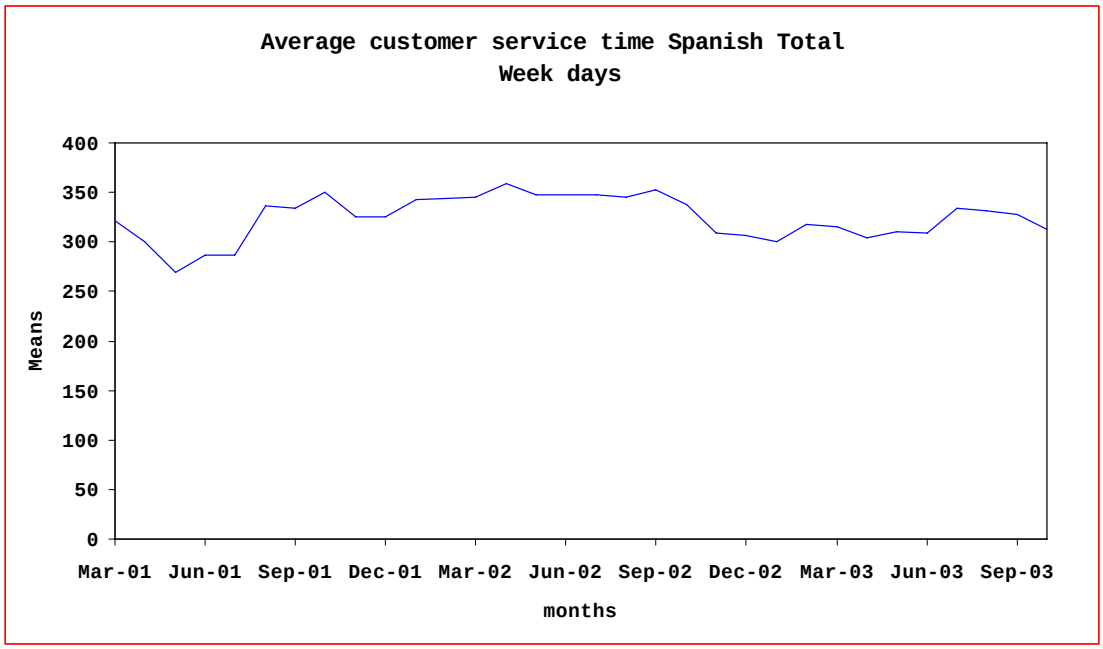


Figure 132 : Average customer service time for Spanish calls



8.2 Daily dynamics of number of agents

All agents that work during a specific day can have their shifts divided according to their status into the following types:

- Long Break – lunch-break about 60 min
- Medium Break – coffee-break about 15 min
- Short Break – short break about 2 min
- Incoming Call – agents who are providing service to incoming calls
- Outgoing Call – agents who are themselves calling customers
- Total – all agents that belong to the five previous types

Note that in this list some agents are not taken into account. For example, agents who are currently idle and available for service of incoming calls. Note that this information is available in the call center of the IL Telecom company (see [3]).

The status of agents is important since it provides information on agents who are physically inside the call center but cannot provide service to incoming calls (because they are on a break or conducting outgoing calls).

Figure 133 presents the distribution of different types of agents per hours for Retail calls in November, 2002. We observe that agents on break constitute a significant part of agents who provide service to incoming calls.

Figure 133 : Agent status for Retail calls (Nov-02)

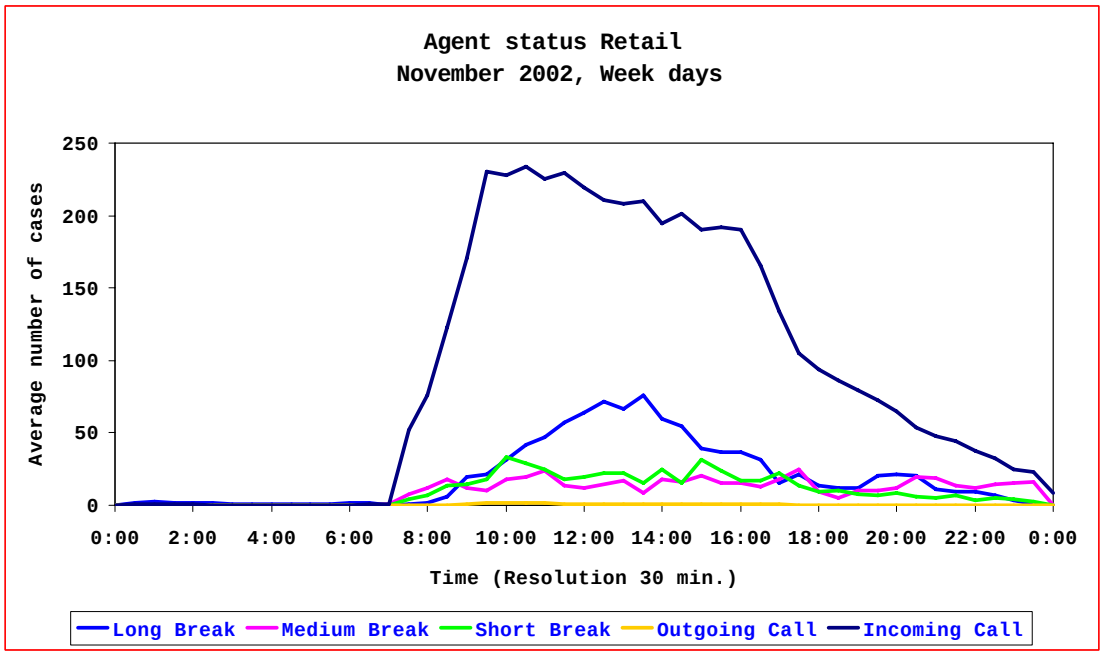


Figure 134 describes the average number of total agents, as well as those on incoming and outgoing calls during weekdays in May, 2001. Note the peak of incoming calls that takes place at 11am. Then, after a light decrease until 12pm, it stabilizes and decreases again after 4pm. In addition, we observe that the two patterns (total and incoming calls) are similar. Note also that number of outgoing calls is very small. The patterns for other months and other customer types are similar.

Figure 134 : Agent status for Retail calls (May-01)

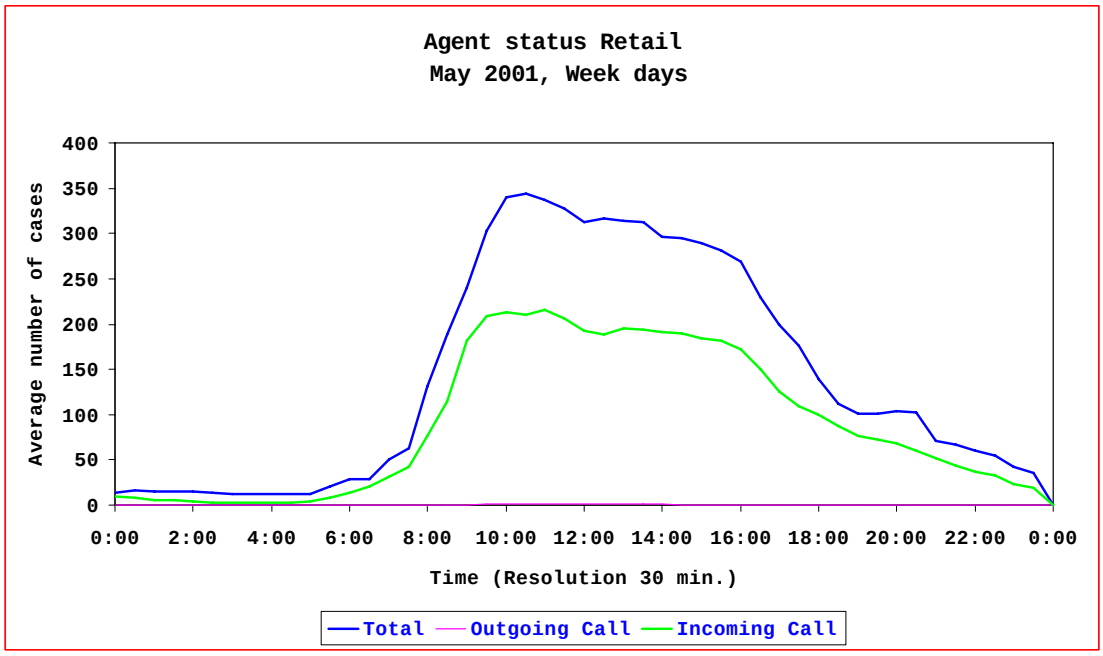


Figure 135 : Arrival rate of Retail calls (May-01)

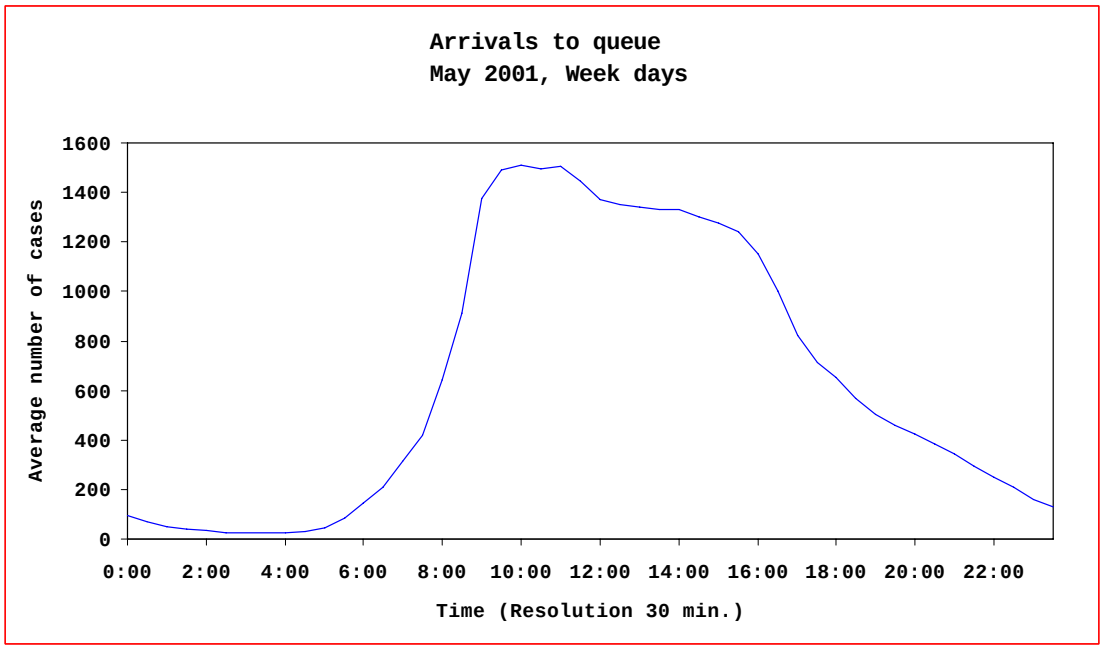


Figure 135 shows arrival rate of Retail calls in May, 2001. We observe that the behavior of the arrival rate is similar to the behavior of the number of agents that serve incoming calls.

9 Survival Analysis

In this section we study relation between patience and average waiting time and we see that this relation can be different for different customer types.

Waiting experience can be summarized by:

- Patience τ, defined as the time that a customer is willing to wait;
- Offered waiting time V, defined as the time that this customer would have to wait given that her or his patience is infinite;
- Waiting time $W_q = \min\{\tau, V\}$, defined as actual waiting time of the customer.

If the offered wait exceeds the customer's patience time, the call is then abandoned (Ab), otherwise the customer awaits service. Note that we observe patience times of only customers that abandon the queue. If a customer gets service, we deduce that his patience τ is larger than the actual wait W. This is classical example of a *censoring* problem.
Techniques for analyzing censored data have been developed within the well-established statistical branch of Survival Analysis.

If we assume that τ is $\exp(\theta)$ distributed, then we can use the Kaplan-Meir estimator of the survival function of patience times:

$$P\{Ab\} = \theta E[W_q]$$

Then we can use the following estimator for average patience:

$$\frac{E[W_q]}{P\{Ab\}}$$

Table 22, Table 23 and Table 24 present estimate of average patience per month.

Table 22: Estimate of average patience, 2001

Date	Mar-01	Apr-01	May-01	Jun-01	Jul-01	Aug-01	Sep-01	Oct-01	Nov-01	Dec-01
Retail	409.03	432.53	420.85	345.45	430.3	444.65	433.81	385	353.09	554.37
Premier	203.4	309.77	281.36	230.11	194.17	235.54	227.23	251	228.9	268.81
Business	99.597	245.06	300.66	205.15	159.24	199.3	265.5	354.8	274.68	342.17
Platinum	350.44	410.4	350.93	337.73	333.85	339.01	481.52	364.6	472.73	430.52
Consumer Loans	551.09	648.75	722.87	531.14	789.99	909.91	883.09	996.9	925.43	893.37
Online Banking	514.83	665.14	687.65	739.8	607.35	516.82	467.73	483.8	460.5	417.34
EBO	1156.6	1142.2	1076.4	1034.6	1063.9	902.93	1102.9	1133	1083.9	1095.5
Telesales	198.07	321.47	229.63	251.46	241.35	248.18	265.21	934.4	282.41	258.01
Spanish	654.39	602.51	467.95	383.22	425.19	515.57	273.58	273.9	243.08	425.28
CCO								2306	1788.4	1551.4
Acquired Bank	687.87	658.96	636.96	689.8	742.46					

Table 23 : Estimate of average patience, 2002

Date	Jan 02	Feb 02	Mar 02	Apr 02	May 02	Jun 02	Jul 02	Aug 02	Sep 02	Oct 02	Nov 02	Dec 02
Retail	626.25	456.91	454.59	289.32	346.76	391.77	485.67	424.18	535.85	404.71	390.73	513.63
Premier	281.78	235.15	290.20	194.95	192.26	163.26	214.21	212.70	278.63	224.69	284.55	262.14
Business	345.51	270.20	234.43	272.66	273.62	318.97	371.83	354.11	370.82	297.48	361.77	359.74
Platinum	315.75	278.06	204.87	192.22	193.26	243.98	234.32	239.52	260.79	241.39	268.26	225.42
Consumer Loan	791.55	662.77	606.92	533.80	570.99	558.08	665.99	693.52	760.40	703.40	1104.20	939.73
Online Banking	430.15	428.55	444.25	361.15	352.12	365.70	409.06	555.54	679.63	634.11	714.57	737.22
EBO	983.99	937.11	1039.01	1010.26	969.03	1068.48	931.47	893.92	1025.17	984.30	1091.35	950.51
Telesales	248.58	314.57	311.85	285.49	214.27	181.47	418.69	388.06	469.00	499.29	481.96	253.39
Spanish	841.53	1101.92	554.34	842.91	1175.07	1236.12	694.63	581.32	410.93	373.30	464.10	356.39
Case Quality							231.26	110.09	95.34	105.34	111.29	108.04
Priority Service					185.27	163.76	99.26	95.88	102.98	94.03	104.07	96.74
AST											857.41	946.21
CCO	1762.13	810.00									1931.65	1761.33
Brokerage											908.84	1109.35
Mortgage											199.90	100.26
BPS											1585.90	1216.26

Table 24 : Estimate of average patience, 2003

Type	Jan-03	Feb-03	Mar-03	Apr-03	May-03	Jun-03	Jul-03	Aug-03	Sep-03	Oct-03
Retail	580.10	685.52	593.88	569.49	583.22	581.54	584.19	621.00	585.11	506.20
Premier	286.58	260.17	198.16	163.80	191.00	213.94	238.30	246.51	236.04	204.89
Business	392.51	340.54	365.72	326.77	377.06	363.91	404.10	323.56	284.73	311.71
Platinum	238.67	189.24	239.29	215.27	253.56	245.92	469.25	242.51	234.06	229.24
Consumer Loan	1102.57	1053.28	724.43	530.43	745.54	871.46	884.38	785.17	654.91	582.80
Online Banking	680.49	618.32	493.37	467.82	430.68	518.81	504.96	504.94	654.45	652.04
EBO	763.76	904.63	750.91	627.38	735.01	939.54	706.30	781.18	456.03	645.90
Telesales	412.11	449.02	374.88	305.81	422.92	450.08	540.82	507.94	323.51	408.36
Spanish	358.65	757.38	538.49	639.59	587.49	354.60	458.21	363.19	341.66	303.67
Case Quality	105.06	102.48	106.74	102.80	105.24	103.96	112.91	109.90	127.77	94.62
Priority Service	86.95	93.19	97.68	98.55	103.75	103.89	132.43	117.41	112.73	121.44
AST	908.55	759.84	953.39	997.10	949.80	891.97	921.18	931.26	971.79	916.68
CCO	409.49	341.44	224.53	239.45	203.79	170.64	517.71	561.90	244.54	157.21
Brokerage	1029.88	646.99	623.87	793.61	764.11	885.68	1135.67	871.98	1029.46	802.23
Mortgage	137.12									
BPS	1268.02	1185.08	1245.92	1202.21	1198.85	1451.87	1366.22	1455.96	1377.48	1449.60

Main observation:

- For some types patience estimate is very unstable. For example: for Consumer Loans calls it is equal to 1104 in November 2002 and to 530 in April 2003; for Telesales calls it is equal to 934 in October 2001 and to 181 in June 2002; for Spanish it is equal to 243 in November 2001 and to 1175 in May 2002.
- Patience is lowest and relatively stable (about 110) for Case Quality calls.
- Patience is large for BPS, Brokerage, AST and EBO (in 2001, 2002).
- Some outliers are observed. For example: September 2003 for EBO calls; February 2002 and May 2002 for Spanish calls.

- Patience of Business calls is larger than the one of Platinum calls in 2002, 2003. Recall that Platinum is the high priority version of Business. Patience of Retail calls is larger than the one of Premier calls. Recall that Premier is the high priority version of Retail.

Now we present graphs of survival estimate and hazard rate for some types of calls.

Figure 136 shows survival estimate for Business calls in July 2001 and September 2002. We observe that average waiting time in September 2002 is ten times larger than in April 2001. However, patience is identical in both months.

Figure 136 : Survival estimate for Business calls

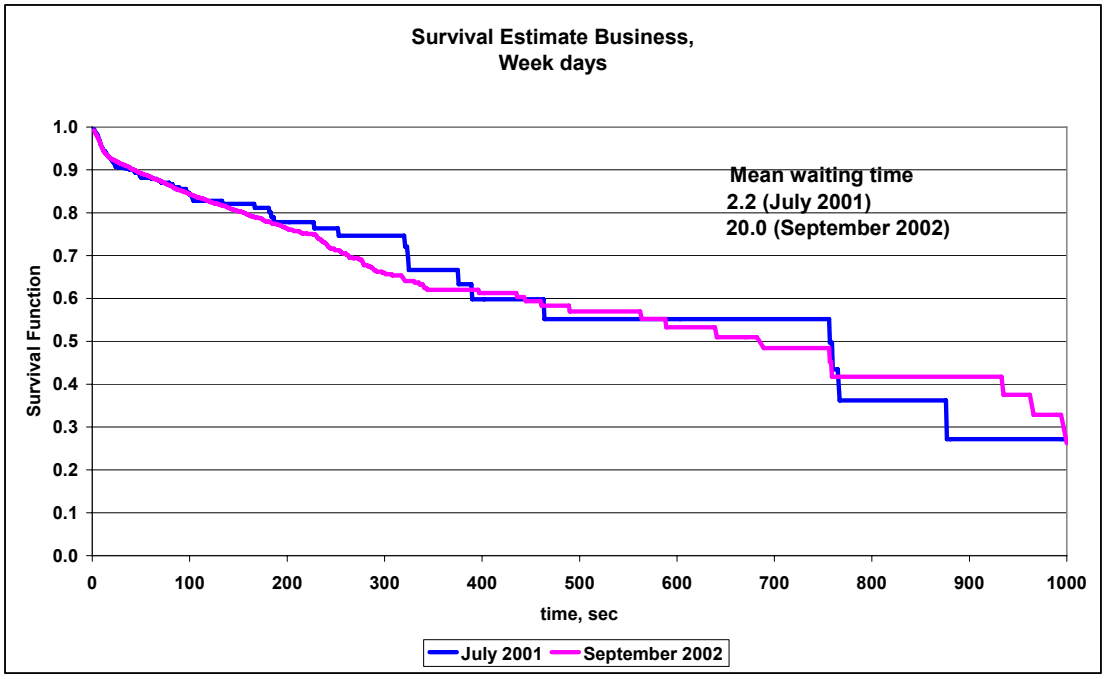


Figure 137 presents survival estimate for Telesales calls in October 2001 and June 2002. We observe a very big difference between average waiting times in October 2001(115.9) and in June 2002 (2.0). Note that less than 60 seconds two patterns coincide; and after 60 seconds the patience in October 2001 is larger. We observe small peak of hazard rate on 46 seconds of wait on October 2001 and significant peak on 73 seconds of wait on June 2002 (see Figure 138).

Figure 137 : Survival estimate for Telesales calls

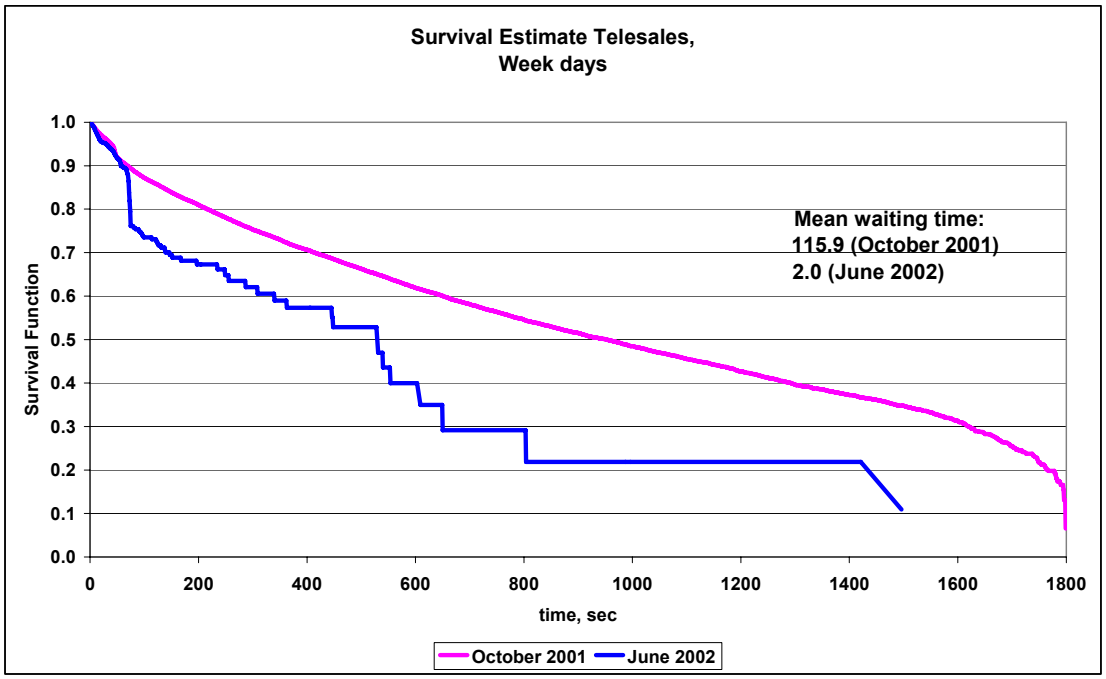


Figure 138 : Hazard rate for Telesales calls

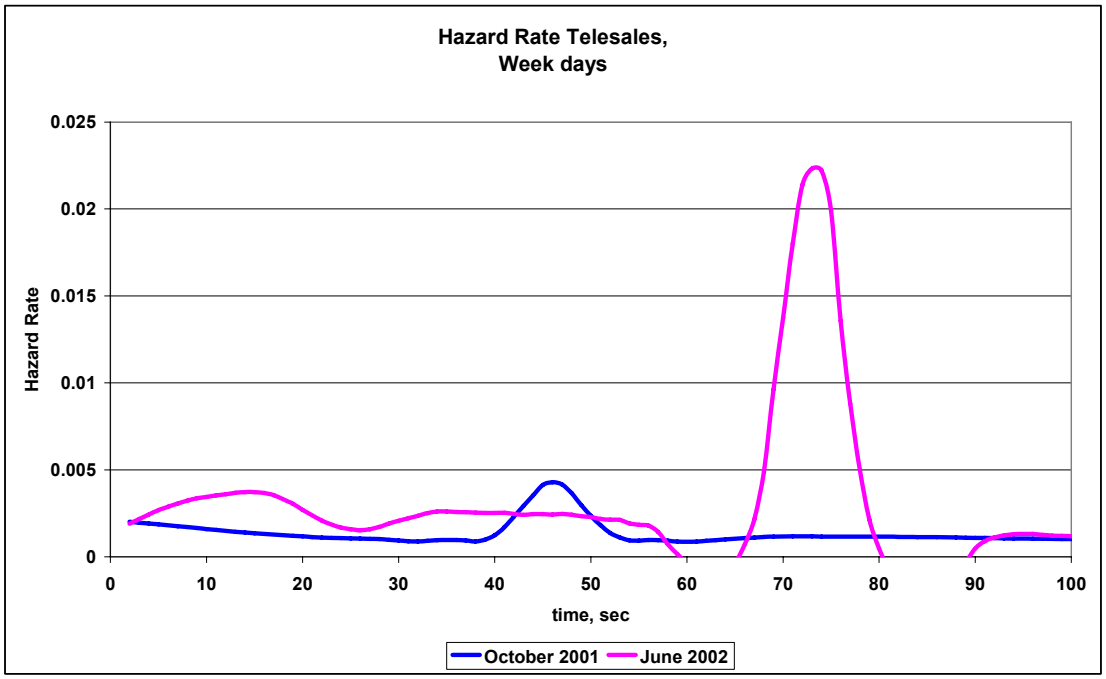


Figure 139 displays survival estimate for Telesales calls in June 2001, February 2003 and June 2003. We observe that three patterns are similar for the time values under 47 seconds; and after 47 seconds patience in June 2001 significant decrease. It can be explained by peak of waiting time took place on 47 seconds (see Figure 140).

Figure 139 : Dynamic of survival estimates for Telesales calls

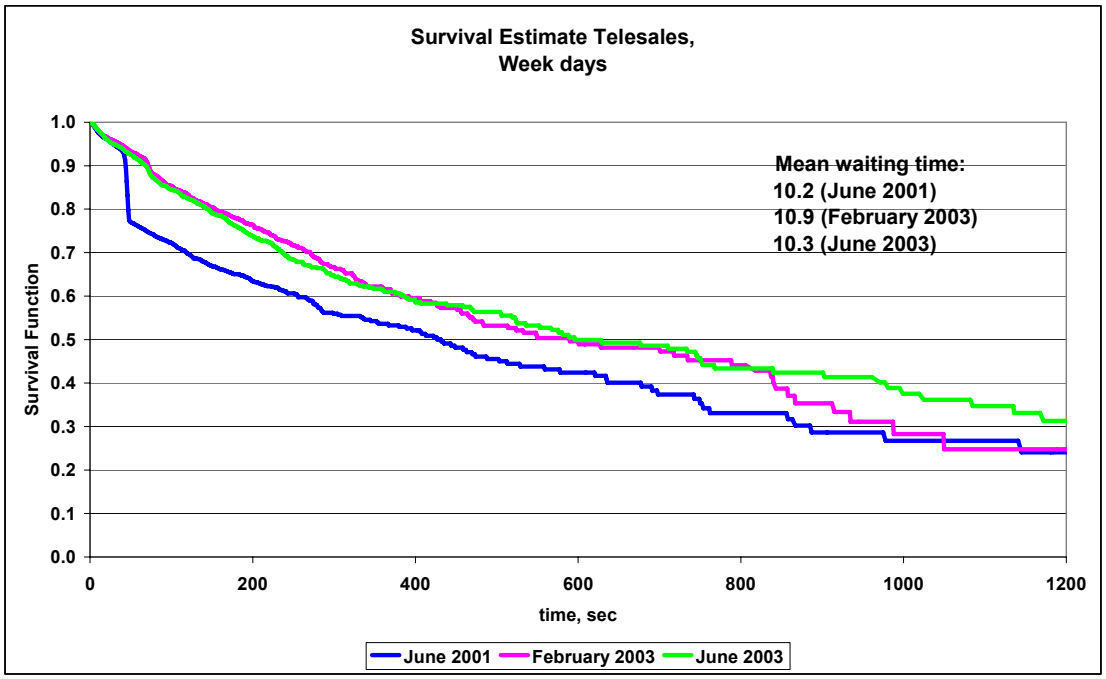


Figure 140 : Histogram of waiting time for Telesales calls (Jun-01)

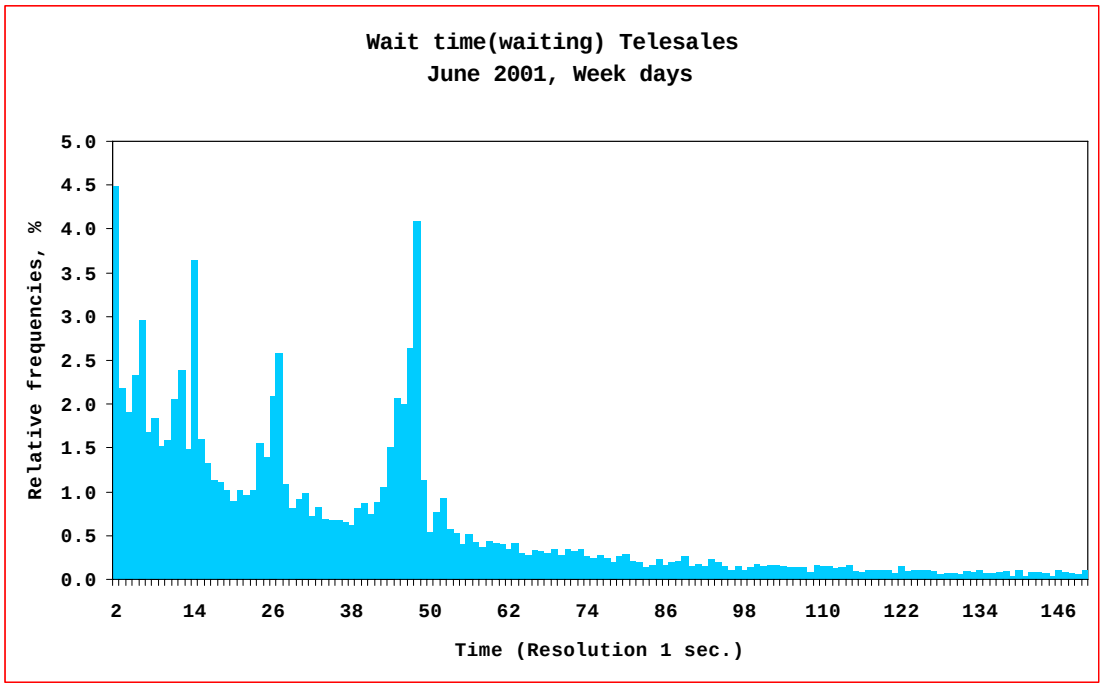


Figure 141 displays survival estimate for Spanish calls in August 2001, February 2002 and September 2003. We observe a big difference between average waiting times in the tree months. Note that the patience have a similar patterns in August 2001 and September 2003 for the time values under about 230 seconds; and the two patterns are similar in February 2002 and September 2003 between about 270 and 370 seconds.

Figure 141 : Survival estimate for Spanish calls

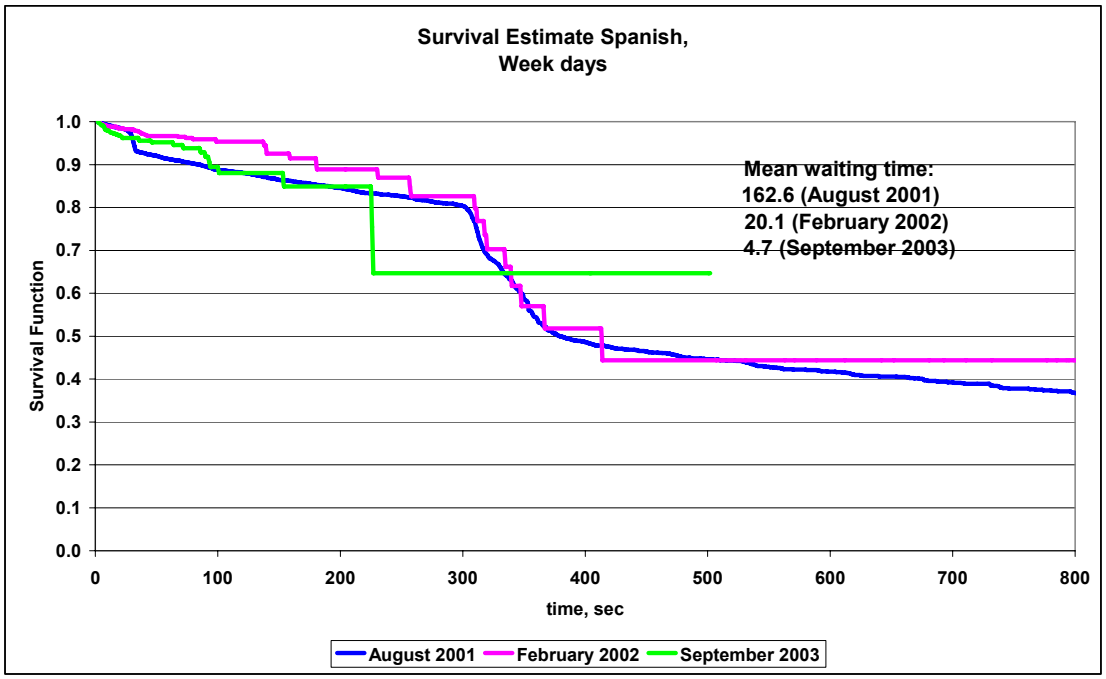


Figure 142 presents survival estimate for CCO calls in December 2002 and June 2003. Note large difference between two patterns that seem to be related to difference in average waiting times (waiting time in December 2002 is about 40 times larger than in June 2003). It can be explained by significant difference between arrival rates in considered months (see Figure 143). Figure 144 displays hazard rate for CCO calls. Note that hazard rate in December 2002 remains stable during all considered time period.

Figure 142 : Survival estimate for CCO calls

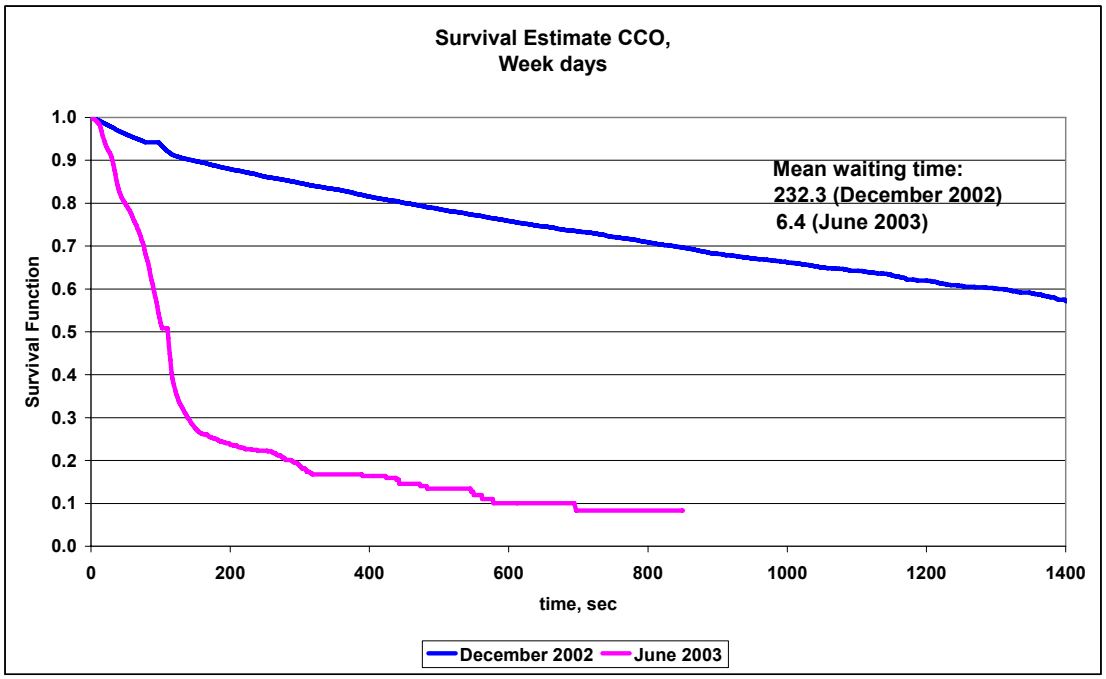


Figure 143 : Arrival rate for CCO calls

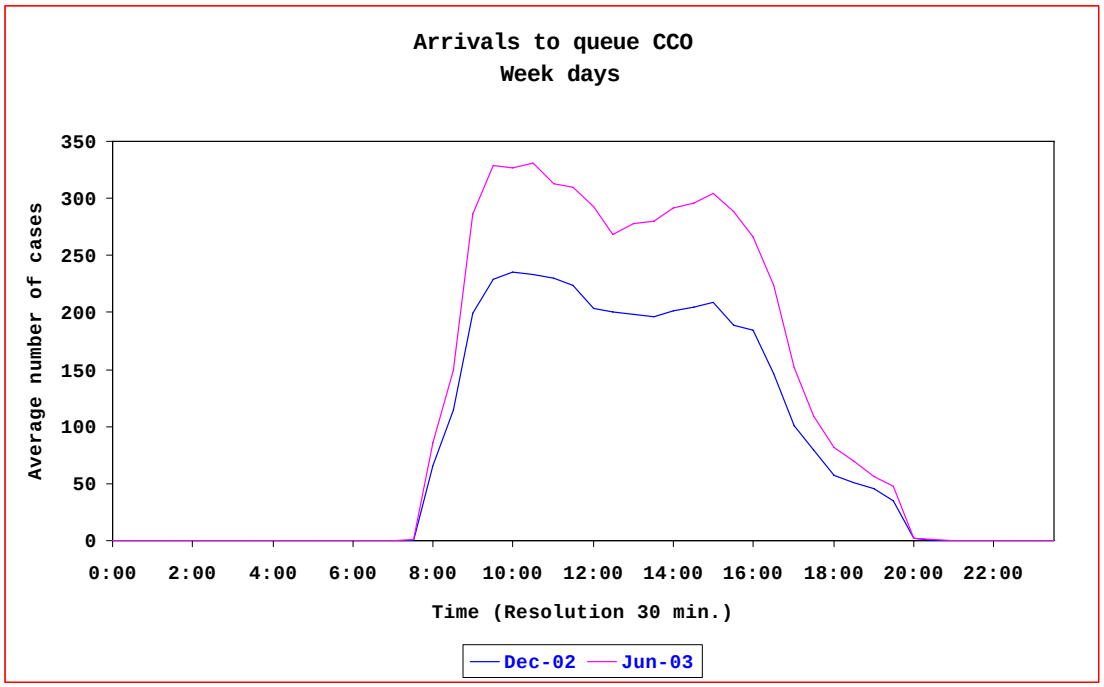


Figure 144 : Hazard rate for CCO calls

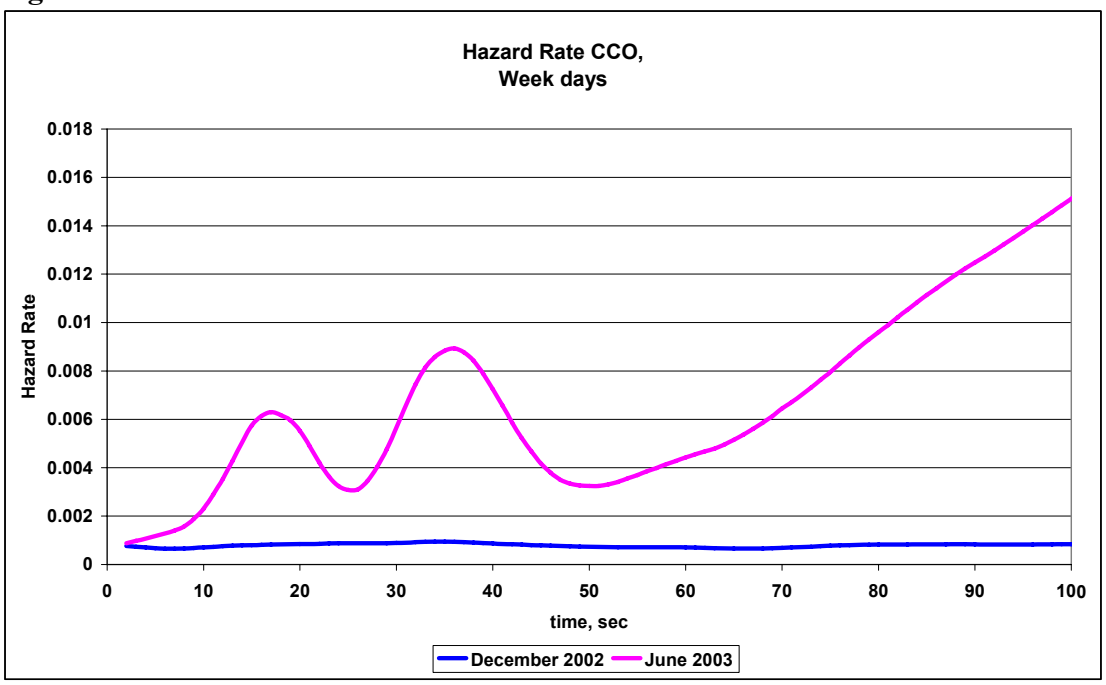


Figure 145 displays survival estimate for CCO calls in three months: January 2003, February 2003 and March 2003. Note that average waiting time decrease from 14.7 in January 2003 to 8.5 in March 2003. Smallest patience matches to smallest average waiting time (green line). Largest patience matches to largest average waiting time for the time values under 300 seconds of wait (blue line); then patience in January coincides with the one in February for the time values under about 375 seconds; and then February patience becomes larger.

Figure 145 : Dynamic of survival estimates for CCO calls

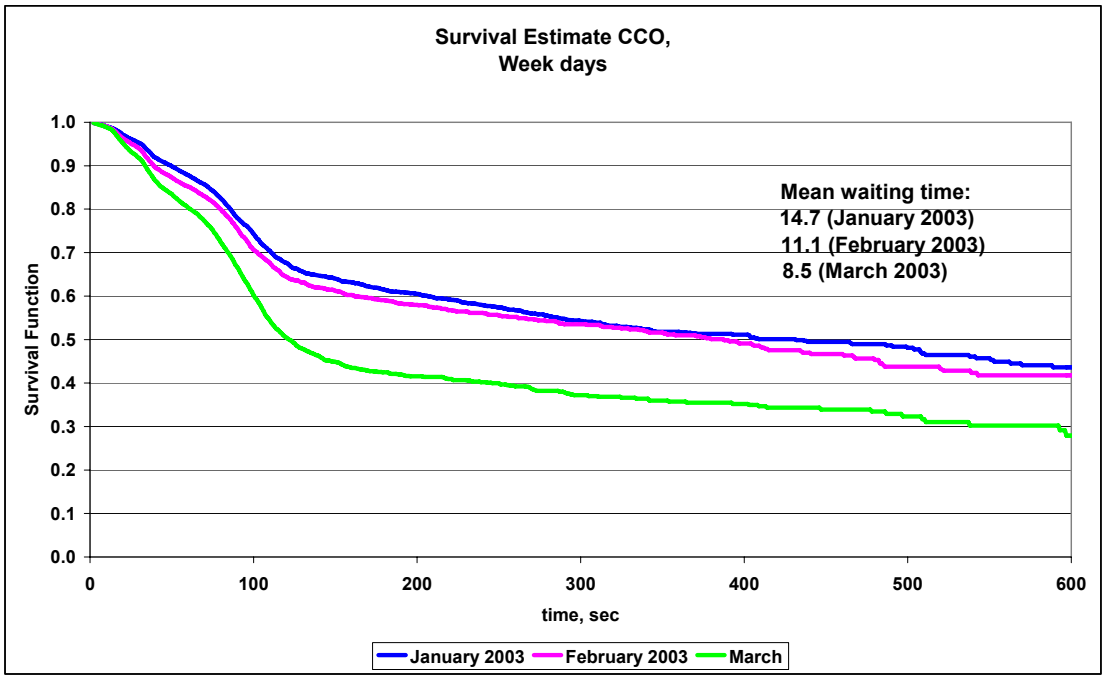


Figure 146 shows survival estimate for Consumer Loans calls in June 2001, October 2001 and February 2003. We observe that less than 80 seconds of wait three patterns coincide; and after 80 seconds patience in February 2003 becomes largest. Note that the average waiting time is smallest in June 2001.

Figure 146 : Survival estimate for Consumer Loans calls

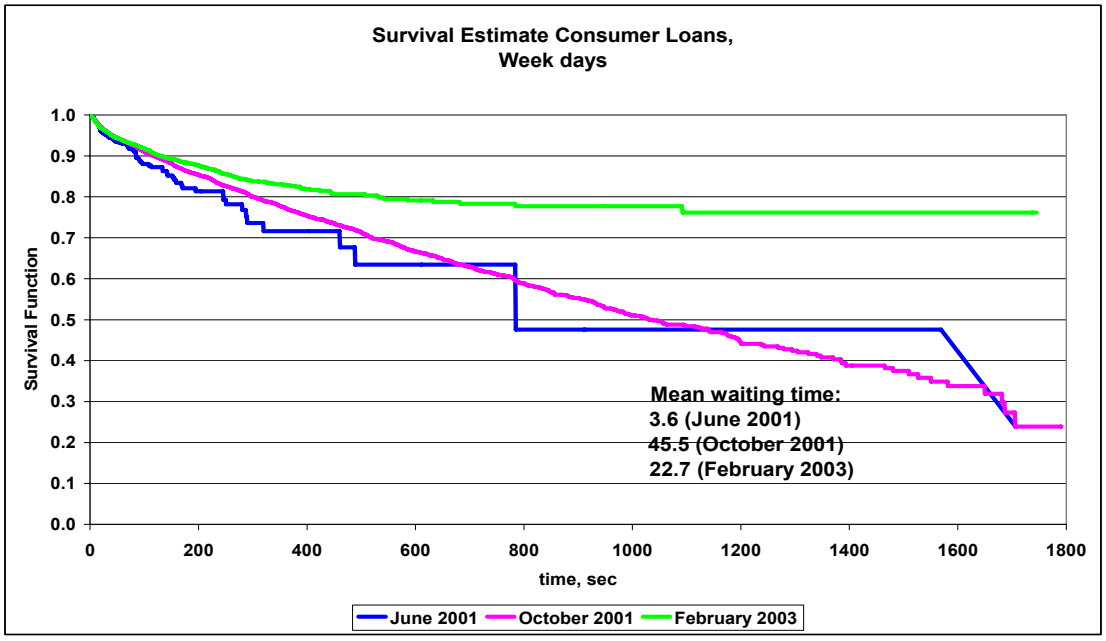


Figure 147 and Figure 148 present survival estimate and hazard rate for Brokerage calls in December 2002, March 2002 and July 2003. Note identical patience for the time values under 600 seconds in December 2002 and July 2003 (see Figure 147), however, the average waiting time in December 2002 is five times larger than in July 2003. Average waiting time and patience are smallest in March 2003.

Figure 147 : Survival estimate for Brokerage calls

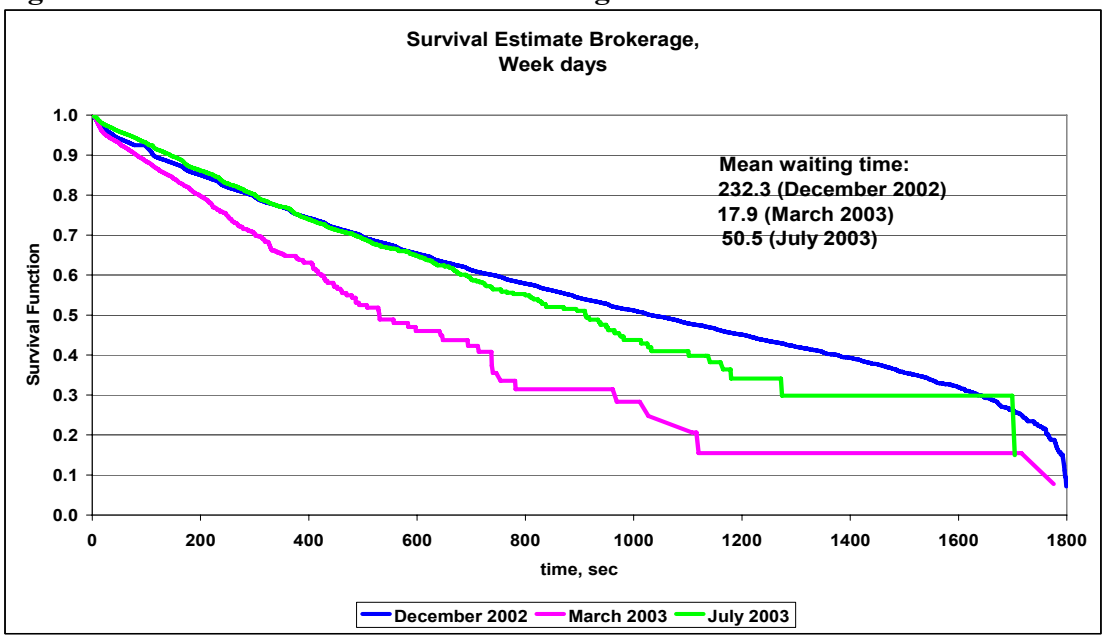


Figure 148 : Hazard rate for Brokerage calls

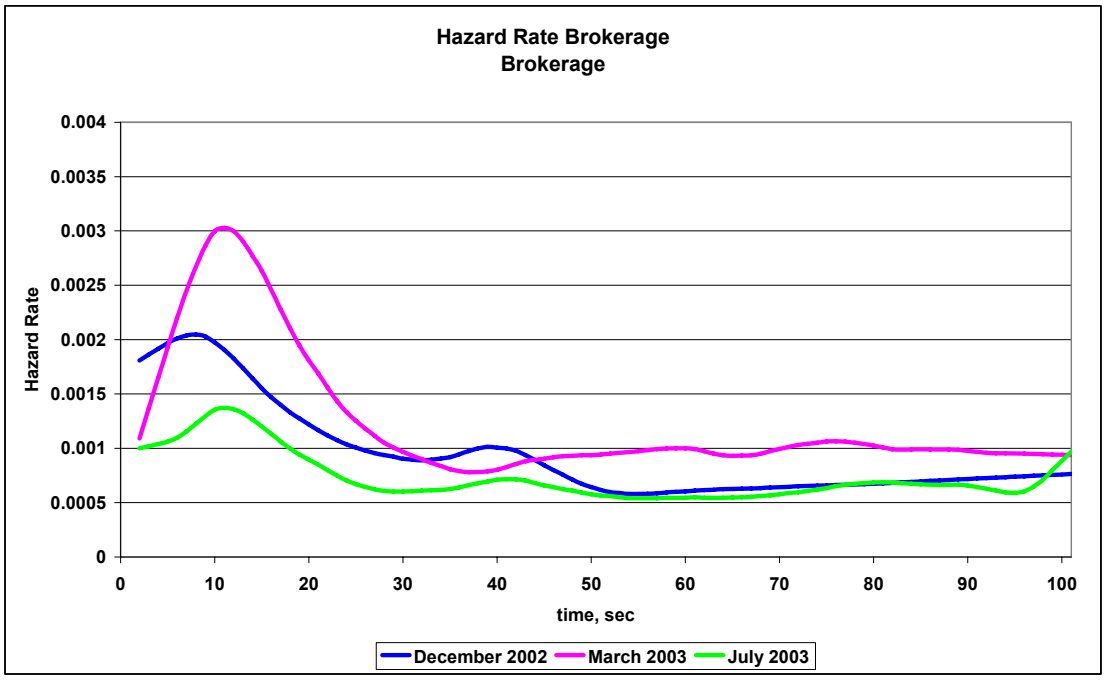


Figure 149 displays survival estimate for Acquired Bank in April and July 2001. We observe identical patterns of patience. Note that average waiting time in July 2001 is four times larger than in April 2001.

Figure 149 : Survival estimate for Acquired Bank calls

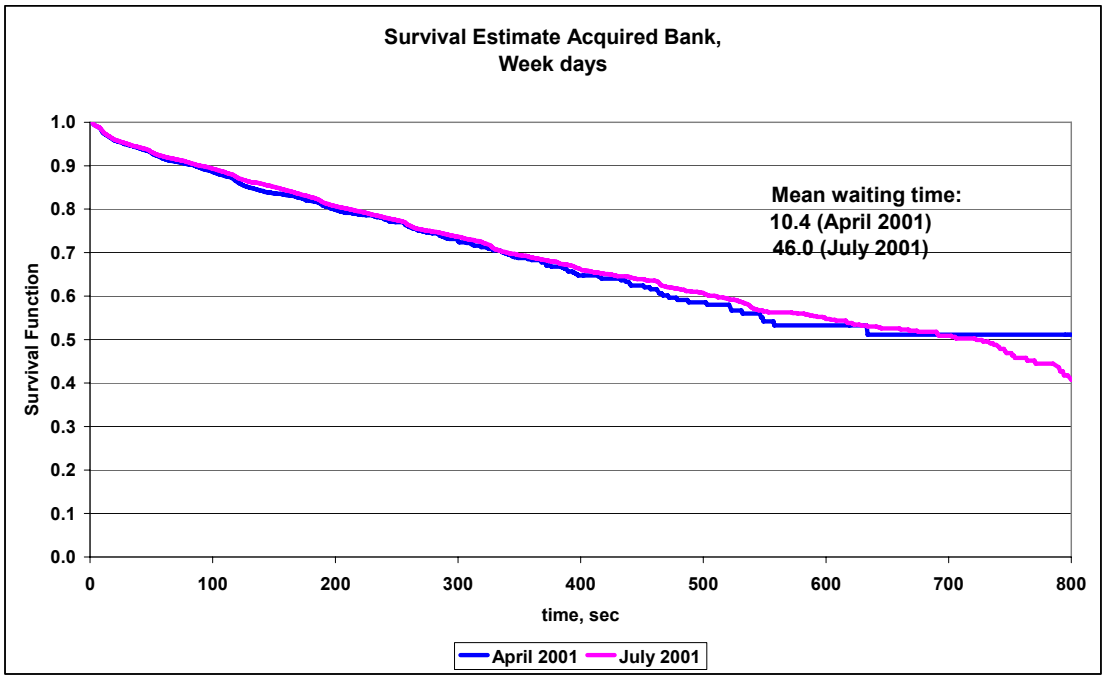


Figure 150 presents survival estimate for CCO calls in four months: from January 2002 until April 2002. Note that average waiting time decreases from 18.9 in January to 10.0 in April. However, four patterns almost coincide for the time values under 142 seconds.

Figure 150 : Dynamic of survival estimates for EBO calls

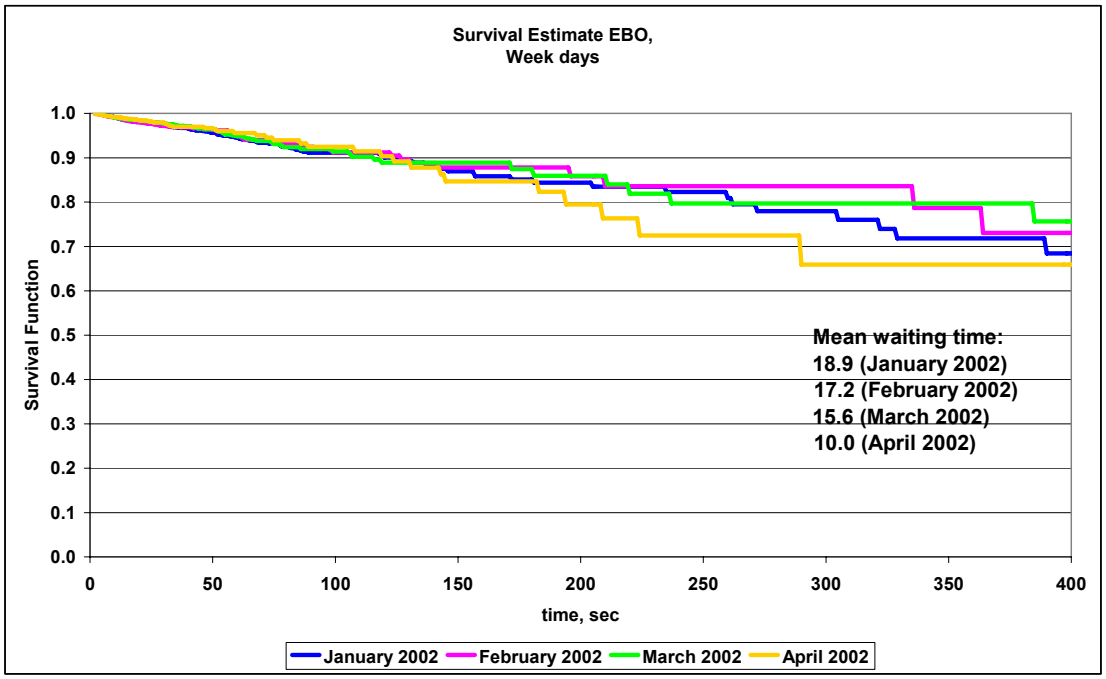


Figure 151 and Figure 152 show survival estimate and hazard rate for Business and Platinum calls in June 2001. Note that patience of Platinum is larger than the one of Business. Recall that Platinum is the high priority version of Business.

Figure 151 : Survival estimate for Business and Platinum calls (Jun-01)

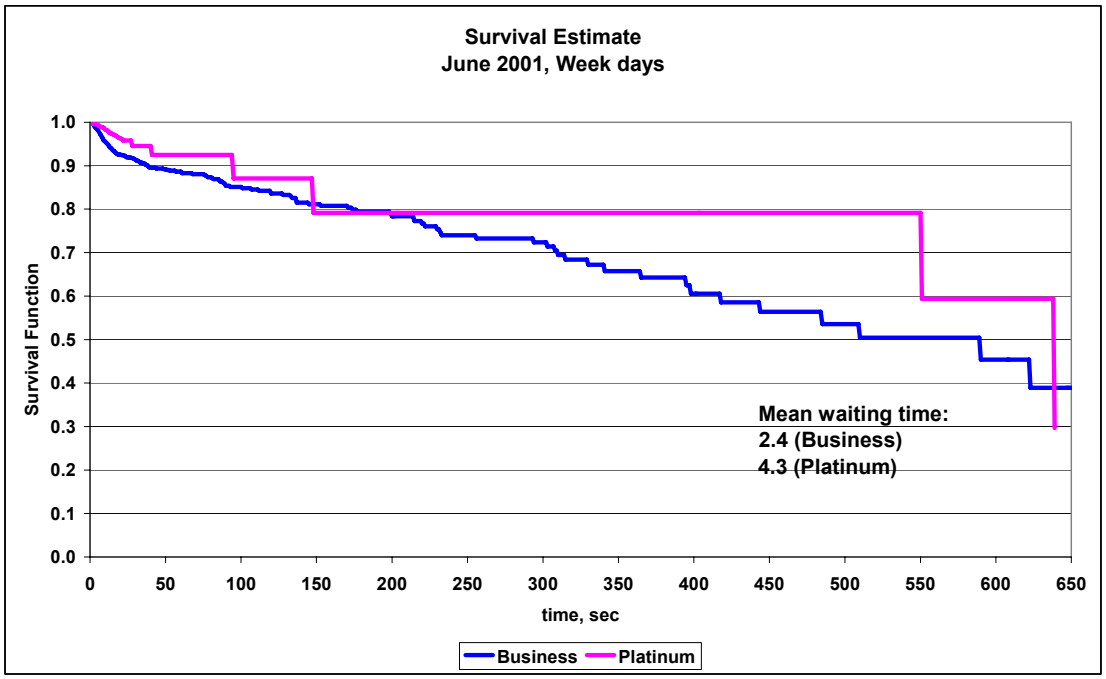


Figure 152 : Hazard rate for Business and Platinum calls (Jun-01)

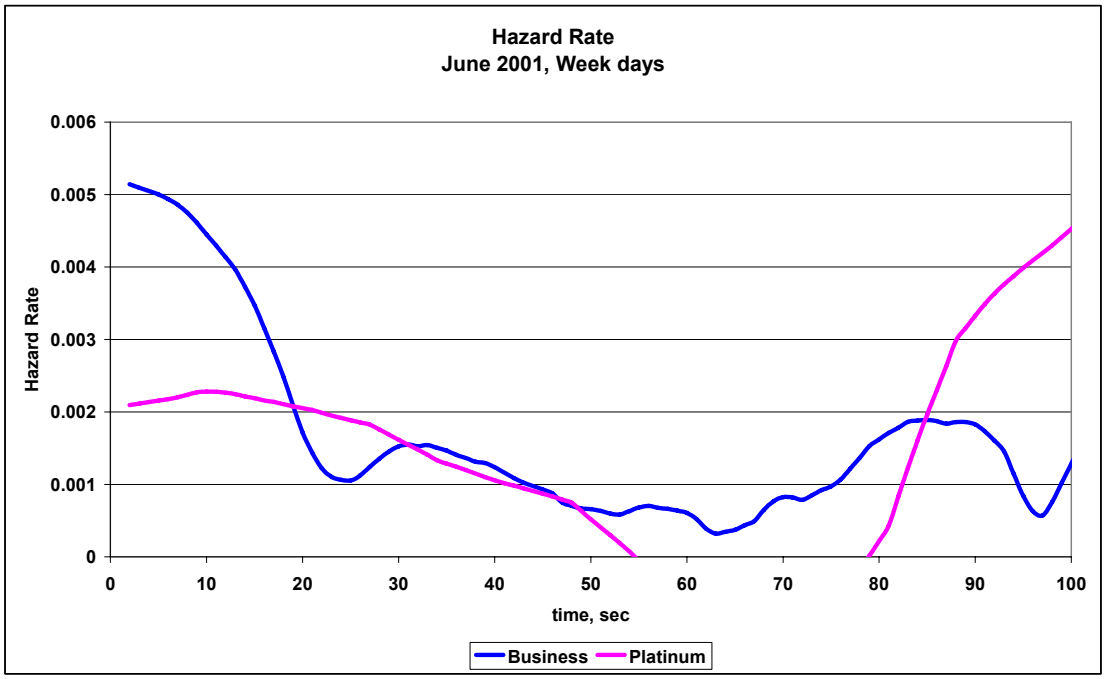


Figure 153 and Figure 154 present survival estimate and hazard rate for Retail and Premier calls in September 2003. We observe that patience of Retail is larger than the one of Premier. Recall that Premier is the high priority version of Retail.

Figure 153 : Survival estimate for Retail and Premier calls (Sept-03)

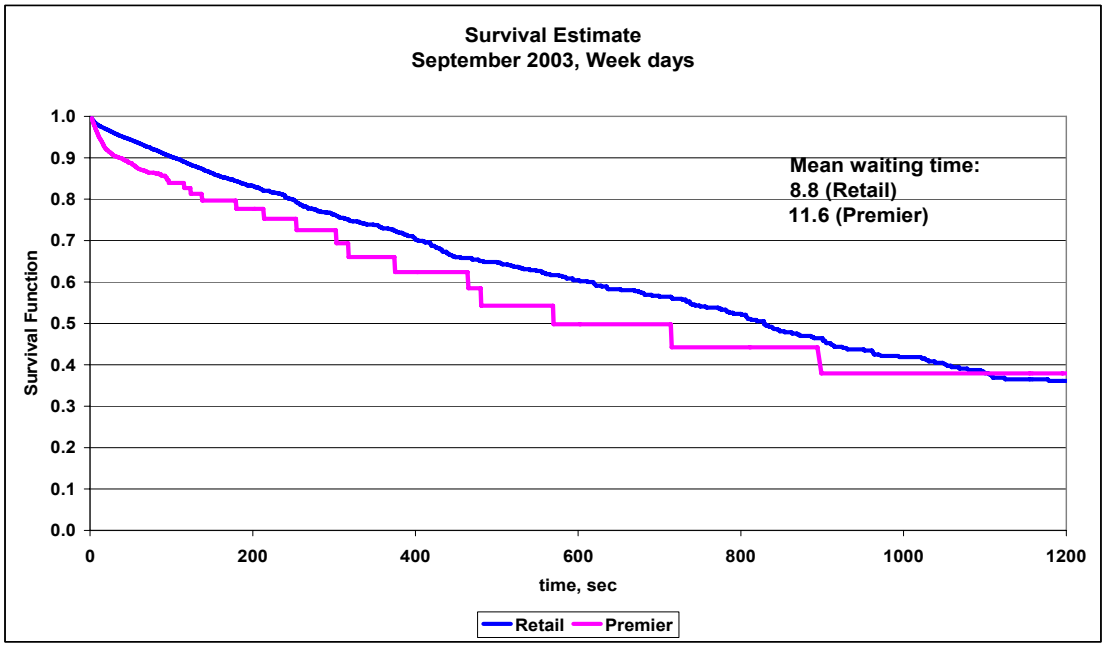


Figure 154 : Hazard rate for Retail and Premier calls (Sept-03)

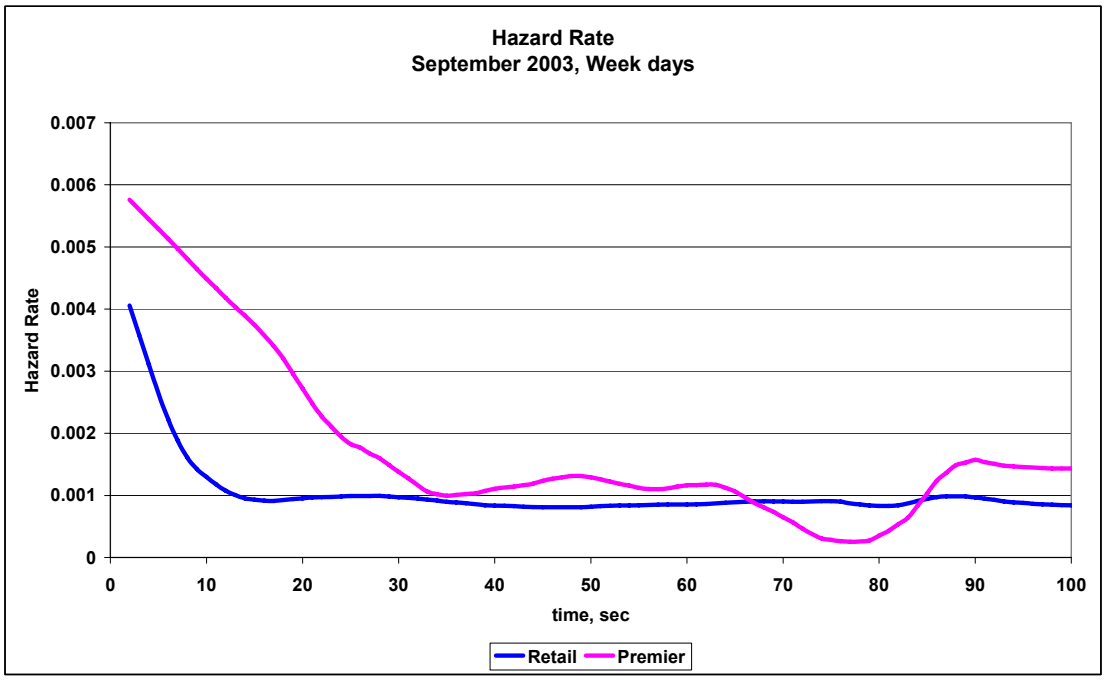


Figure 155 presents survival estimate for some customer types: Business, Consumer Loans and Case Quality in December 2002. Note that average waiting times for considered types are similar (13.6 for Business calls, 14.0 for Consumer Loans calls and 13.5 for Case Quality calls). We observe that patience of Case Quality customers (green line) becomes much less than patience of other customer types after 22 seconds of wait. It can be explained by peaks of waiting time took place till 22 seconds (see Figure 156). Patience of Consumer Loans customers is largest (violet line).

Figure 155 : Survival estimate for some call types (Dec-02)

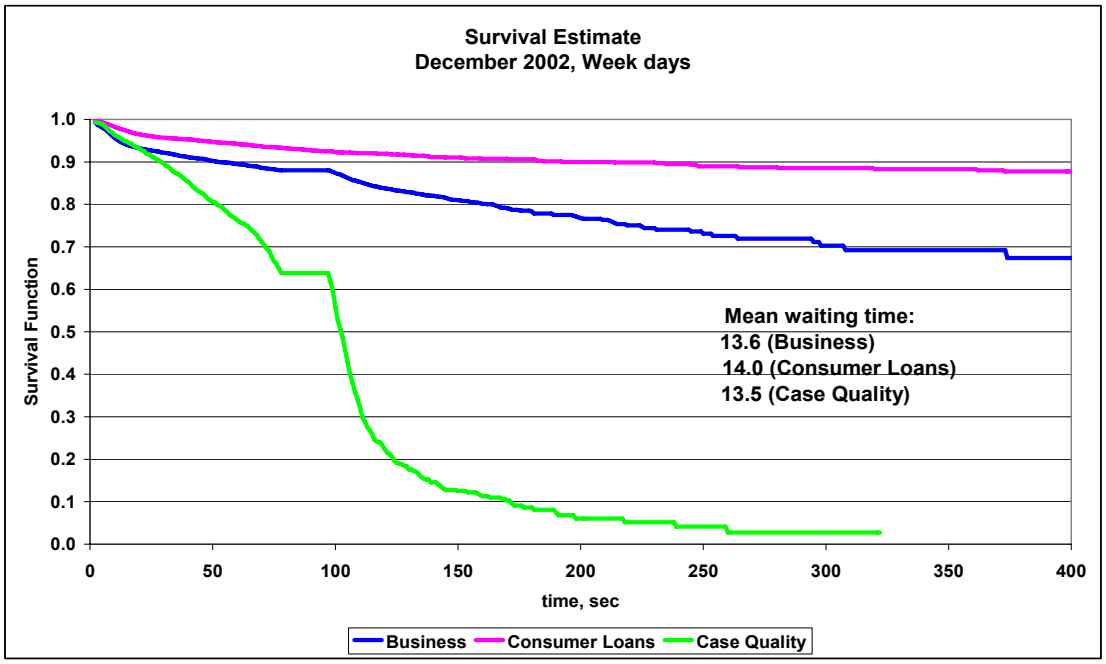


Figure 156 : Histogram of waiting time for Case Quality calls (Dec-02)

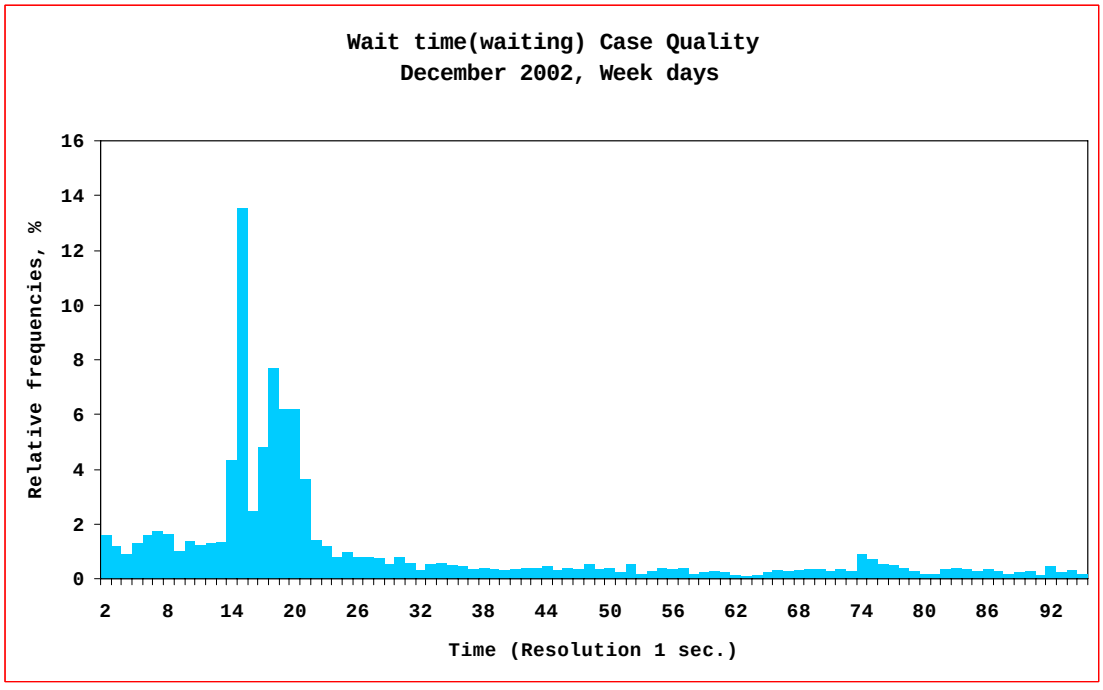


Figure 157 shows survival estimate for prevalent customer types: Retail, Business and Telesales in September 2001. Note that average waiting times for considered types are similar (3.5 for Retail calls and 3.7 for Business and Telesales calls). We observe that patience of Retail customers is largest (blue line). Patience of Telesales customers is larger than the one for Business for the time values under 50 seconds of wait; and after 50 seconds patience of Business customers becomes larger (violet and green lines).

Figure 157 : Survival estimate for some call types (Sept-01)

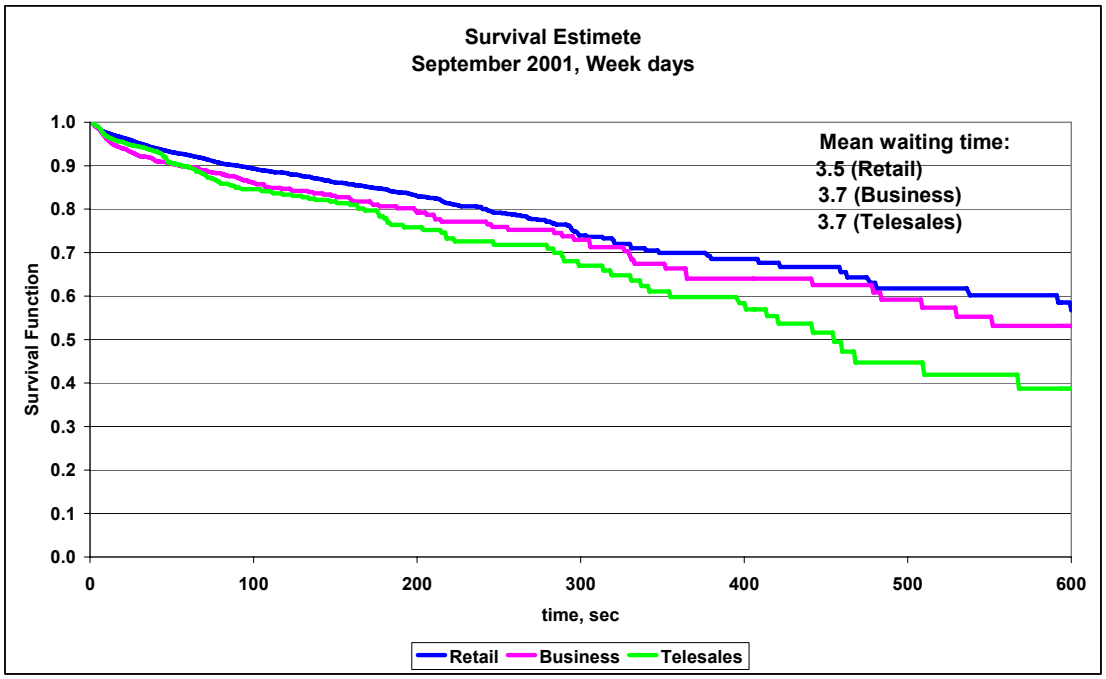
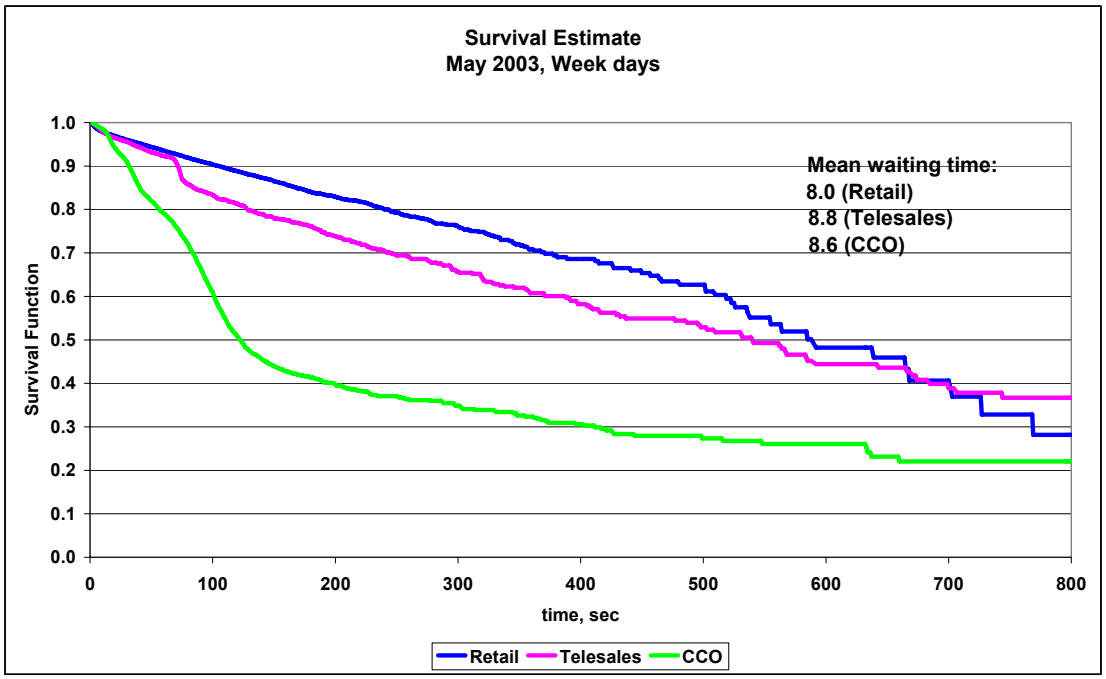


Figure 158 presents survival estimate for some customer types: Retail, Telesales and CCO in May 2003. Note that average waiting times for considered types are similar (8.0 for Retail calls, 8.8 for Telesales calls and 8.6 for CCO calls). We observe that three patterns coincide for the time values under 15 seconds; and after 15 seconds patience of CCO customers becomes smallest (green line). Note also that less than 60 seconds of wait patience of Retail and Business customers are similar; after 60 seconds patience of Telesales becomes smaller (violet line).

Figure 158 : Survival estimate for some call types (May-03)



10 Basic statistics

Table 25: Monthly call counts (% out of yearly total)

Year 2001

month	Total # of arriving calls	%
March2001 (26/03-31/03)	1,312,225	2.34%
April2001	5,354,743	9.53%
May2001	4,820,670	8.58%
June2001	5,592,138	9.96%
July2001	6,729,583	11.98%
August2001	6,262,038	11.15%
September2001	5,922,556	10.54%
October2001	6,865,859	12.22%
November2001	6,631,067	11.80%
December2001	6,682,635	11.90%
Total 2001	56,173,514	100%
Average per month 2001	**6,105,817**	**10.87%**

Year 2002

month	Total # of arriving calls	%
January2002	6,964,811	8.46%
February2002	6,442,941	7.83%
March2002	6,908,412	8.39%
April2002	7,163,178	8.70%
May2002	6,857,077	8.33%
June2002	6,721,987	8.17%
July2002	7,105,554	8.63%
August2002	7,098,799	8.62%
September2002	5,795,582	7.04%
October2002	7,029,396	8.54%
November2002	6,446,038	7.83%
December2002	7,785,089	9.46%
Total 2001	82,318,864	100%
Average per month 2002	**6,859,905**	**8.33%**

Year 2003

month	Total # of arriving calls	%
January2003	8,340,522	10.48%
February2003	8,403,736	10.56%
March2003	8,606,779	10.82%
April2003	7,081,624	8.90%
May2003	8,047,694	10.12%
June2003	8,178,286	10.28%
July2003	8,477,345	10.66%
August2003	8,135,475	10.23%
September2003	7,745,113	9.74%
October2003 (1/10-26/10)	6,538,536	8.22%
Total 2001	79,555,110	100%
Average per month 2003	**8,084,869**	**10.16%**

Table 26: Handled/unhandled calls (% out of monthly total # calls arrivals to the queue)

Year 2001

month	# calls arrivals to the queue (offered)	handled	% handled	unhandled	% unhandled
March2001 (26/03-31/03)	278,340	269,499	96.82%	8,841	3.18%
April2001	1,148,091	1,126,077	98.08%	22,014	1.92%
May2001	1,071,441	1,055,407	98.50%	16,034	1.50%
June2001	1,228,083	1,210,288	98.55%	17,795	1.45%
July2001	1,569,311	1,544,312	98.41%	24,999	1.59%
August2001	1,421,573	1,398,687	98.39%	22,886	1.61%
September2001	1,241,439	1,227,211	98.85%	14,228	1.15%
October2001	1,461,667	1,430,883	97.89%	30,784	2.11%
November2001	1,317,845	1,302,535	98.84%	15,310	1.16%
December2001	1,290,754	1,269,218	98.33%	21,536	1.67%
Total	**12,028,544**	**11,834,117**	**98.38%**	**194,427**	**1.62%**
Average per moth 2001	**1,307,450**	**1,286,317**	**98.38%**	**21,133**	**1.62%**

Year 2002

month	# calls arrivals to the queue (offered)	handled	% handled	unhandled	% unhandled
Jan-02	1,407,885	1,381,595	98.13%	26,290	1.87%
Feb-02	1,283,705	1,254,821	97.75%	28,884	2.25%
Mar-02	1,373,182	1,340,953	97.65%	32,229	2.35%
Apr-02	1,300,913	1,281,508	98.51%	19,405	1.49%
May-02	1,218,224	1,202,039	98.67%	16,185	1.33%
Jun-02	1,182,492	1,159,417	98.05%	23,075	1.95%
Jul-02	1,336,457	1,303,051	97.50%	33,406	2.50%
Aug-02	1,354,110	1,320,471	97.52%	33,639	2.48%
Sep-02	1,069,983	1,036,895	96.91%	33,088	3.09%
Oct-02	1,294,056	1,258,930	97.29%	35,126	2.71%
Nov-02	1,113,198	1,077,099	96.76%	36,099	3.24%
Dec-06	1,456,581	1,388,237	95.31%	68,344	4.69%
Total	**15,390,786**	**15,005,016**	**97.49%**	**385,770**	**2.51%**
Average per moth 2002	**1,282,566**	**1,250,418**	**97.49%**	**32,148**	**2.51%**

Year 2003

month	# calls arrivals to the queue (offered)	handled	% handled	unhandled	% unhandled
January2003	1,508,497	1,467,000	97.25%	41,497	2.75%
February2003	1,307,342	1,270,903	97.21%	36,439	2.79%
March2003	1,524,002	1,486,669	97.55%	37,333	2.45%
April2003	1,301,752	1,272,639	97.76%	29,113	2.24%
May2003	1,412,498	1,379,132	97.64%	33,366	2.36%
June2003	1,445,121	1,408,032	97.43%	37,089	2.57%
July2003	1,616,095	1,564,755	96.82%	51,340	3.18%
August2003	1,521,577	1,473,751	96.86%	47,826	3.14%
September2003	1,439,791	1,398,239	97.11%	41,552	2.89%
October2003 (1/10-26/10)	1,150,137	1,122,598	97.61%	27,539	2.39%
Total	**14,226,812**	**13,843,718**	**97.31%**	**383,094**	**2.69%**
Average per moth 2003	**1,445,814**	**1,406,882**	**97.31%**	**38,932**	**2.69%**

Table 27: Call counts by waiting status (% out of monthly total # calls arrivals to the queue)

Year 2001

month	# calls arrivals to the queue (offered)	number_of_waiting	% of waiting	no_waiting	% of no waiting
March2001 (26/03-31/03)	278,340	95,868	34.44%	182,472	65.56%
April2001	1,148,091	283,701	24.71%	864,390	75.29%
May2001	1,071,441	202,853	18.93%	868,588	81.07%
June2001	1,228,083	186,584	15.19%	1,041,499	84.81%
July2001	1,569,311	315,663	20.11%	1,253,648	79.89%
August2001	1,421,573	374,236	26.33%	1,047,337	73.67%
September2001	1,241,439	230,015	18.53%	1,011,424	81.47%
October2001	1,461,667	349,303	23.90%	1,112,364	76.10%
November2001	1,317,845	292,060	22.16%	1,025,785	77.84%
December2001	1,290,754	363,163	28.14%	927,591	71.86%
Total	**12,028,544**	**2,693,446**	**22.39%**	**9,335,098**	**77.61%**
Average per month 2001	**1,307,450**	**292,766**	**22.39%**	**1,014,685**	**77.61%**

Year 2002

month	# calls arrivals to the queue (offered)	number_of_waiting	% of waiting	no_waiting	% of no waiting
January2002	1,407,885	427,474	30.36%	980,411	69.64%
February2002	1,283,705	446,367	34.77%	837,338	65.23%
March2002	1,373,182	471,690	34.35%	901,492	65.65%
April2002	1,300,913	303,706	23.35%	997,207	76.65%
May2002	1,218,224	293,131	24.06%	925,093	75.94%
June2002	1,182,492	357,855	30.26%	824,637	69.74%
July2002	1,336,457	514,465	38.49%	821,992	61.51%
August2002	1,354,110	544,654	40.22%	809,456	59.78%
September2002	1,069,983	493,317	46.11%	576,666	53.89%
October2002	1,294,056	462,880	35.77%	831,176	64.23%
November2002	1,113,198	399,210	35.86%	713,988	64.14%
December2002	1,456,581	539,645	37.05%	916,936	62.95%
Total	**15,390,786**	**5,254,394**	**34.14%**	**10,136,392**	**65.86%**
Average per month 2002	**1,282,565**	**437,866**	**34.14%**	**844,699**	**65.86%**

Year 2003

month	# calls arrivals to the queue (offered)	number_of_waiting	% of waiting	no_waiting	% of no waiting
January2003	1,508,497	510,290	33.83%	998,207	66.17%
February2003	1,307,342	493,527	37.75%	813,815	62.25%
March2003	1,524,002	484,332	31.78%	1,039,670	68.22%
April2003	1,301,752	406,154	31.20%	895,598	68.80%
May2003	1,412,498	460,580	32.61%	951,918	67.39%
June2003	1,445,121	513,553	35.54%	931,568	64.46%
July2003	1,616,095	683,105	42.27%	932,990	57.73%
August2003	1,521,577	646,020	42.46%	875,557	57.54%
September2003	1,439,791	541,615	37.62%	898,176	62.38%
October2003 (1/10-26/10)	1,150,137	325,377	28.29%	824,760	71.71%
Total	**14,226,812**	**5,064,553**	**35.60%**	**9,162,259**	**64.40%**
Average per month 2003	**1,445,814**	**514,690**	**35.60%**	**931,124**	**64.40%**

Table 28: Call Counts by service termination

Year 2001

month	Caller Termin	% Caller Term	Agent Termin	% Agent Term	Undeter Termin	% Undeter Term	Transfer	% Transfer	Total
Mar-01 (26/03-31/03)	187,985	69.87%	42,769	15.90%	3,946	1.47%	34,357	12.77%	269,057
Apr-01	791,426	70.38%	181,343	16.13%	14,967	1.33%	136,712	12.16%	1,124,448
May-01	720,735	68.39%	190,042	18.03%	12,647	1.20%	130,436	12.38%	1,053,860
Jun-01	821,643	68.05%	218,230	18.07%	14,343	1.19%	153,279	12.69%	1,207,495
Jul-01	1,038,110	67.39%	272,965	17.72%	20,191	1.31%	209,137	13.58%	1,540,403
Aug-01	943,010	67.60%	250,298	17.94%	19,665	1.41%	181,935	13.04%	1,394,908
Sep-01	827,518	67.70%	223,254	18.26%	15,574	1.27%	155,991	12.76%	1,222,337
Oct-01	962,989	67.42%	256,539	17.96%	19,017	1.33%	189,824	13.29%	1,428,369
Nov-01	883,109	67.86%	231,855	17.82%	17,185	1.32%	169,244	13.00%	1,301,393
Dec-01	879,353	69.35%	218,684	17.25%	16,644	1.31%	153,286	12.09%	1,267,967

Year 2002

month	Caller Termin	% Caller Term	Agent Termin	% Agent Term	Undeter Termin	% Undeter Term	Transfer	% Transfer	Total
Jan-02	967,347	70.06%	227,398	16.47%	17,662	1.28%	168,405	12.20%	1,380,812
Feb-02	896,540	71.52%	189,981	15.16%	16,308	1.30%	150,678	12.02%	1,253,507
Mar-02	959,435	71.62%	197,015	14.71%	17,392	1.30%	165,865	12.38%	1,339,707
Apr-02	917,980	71.64%	185,641	14.49%	15,922	1.24%	161,758	12.62%	1,281,301
May-02	862,535	71.77%	174,731	14.54%	14,141	1.18%	150,360	12.51%	1,201,767
Jun-02	836,938	72.21%	165,319	14.26%	13,891	1.20%	142,953	12.33%	1,159,101
Jul-02	937,007	72.01%	184,057	14.15%	17,273	1.33%	162,878	12.52%	1,301,215
Aug-02	952,189	72.17%	179,909	13.64%	18,261	1.38%	168,946	12.81%	1,319,305
Sep-02	752,977	73.08%	132,983	12.91%	13,686	1.33%	130,759	12.69%	1,030,405
Oct-02	904,102	71.92%	169,804	13.51%	17,401	1.38%	165,812	13.19%	1,257,119
Nov-02	745,608	70.07%	170,404	16.01%	13,803	1.30%	134,343	12.62%	1,064,158
Dec-02	925,285	68.43%	249,730	18.47%	17,293	1.28%	159,789	11.82%	1,352,097

Year 2003

month	Caller Termin	% Caller Term	Agent Termin	% Agent Term	Undeter Termin	% Undeter Term	Transfer	% Transfer	Total
Jan-03	982,878	68.16%	264,878	18.37%	17,427	1.21%	176,773	12.26%	1,441,956
Feb-03	864,300	68.62%	226,212	17.96%	15,263	1.21%	153,729	12.21%	1,259,504
Mar-03	999,051	67.78%	266,642	18.09%	17,766	1.21%	190,508	12.92%	1,473,967
Apr-03	866,712	68.78%	223,023	17.70%	14,414	1.14%	156,032	12.38%	1,260,181
May-03	945,070	69.26%	237,454	17.40%	15,715	1.15%	166,219	12.18%	1,364,458
Jun-03	960,852	69.10%	245,160	17.63%	16,509	1.19%	168,092	12.09%	1,390,613
Jul-03	1,058,864	68.93%	270,531	17.61%	19,610	1.28%	187,248	12.19%	1,536,253
Aug-03	1,007,527	69.00%	254,078	17.40%	19,182	1.31%	179,304	12.28%	1,460,091
Sep-03	965,233	69.91%	230,167	16.67%	17,673	1.28%	167,644	12.14%	1,380,717
Oct-03 (1/1026/10)	781,306	70.47%	181,088	16.33%	13,855	1.25%	132,480	11.95%	1,108,729

Table 29: Average number of agents per month

Year 2001

Date	week days	Saturdays	Sundays
Mar-01	911	375	
Apr-01	972	398	227
May-01	1009	414	241
Jun-01	1034	455	290
Jul-01	1108	533	301
Aug-01	1036	457	255
Sep-01	1010	410	237
Oct-01	1039	383	254
Nov-01	997	433	238
Dec-01	914	417	222
Average	1013	429	252

Year 2002

Date	week days	Saturdays	Sundays
Jan-02	907	436	221
Feb-02	876	406	218
Mar-02	834	380	196
Apr-02	867	318	149
May-02	847	295	127
Jun-02	839	289	128
Jul-02	811	286	123
Aug-02	806	295	126
Sep-02	808	297	130
Oct-02	841	296	135
Nov-02	940	325	148
Dec-02	1087	365	153
Average	872	332	154

Year 2003

Date	week days	Saturdays	Sundays
Jan-03	1154	387	151
Feb-03	1160	385	152
Mar-03	1199	400	157
Apr-03	1181	372	133
May-03	1134	349	136
Jun-03	1116	333	144
Jul-03	1112	327	139
Aug-03	1105	361	148
Sep-03	1144	375	156
Oct-03	1140	358	149
Average	1145	365	146

11 Some queueing science

11.1 Validation of Little's law

Little's law is a conservation law that relates three quantities: arrival rate (λ), waiting time (W) and the number customers in the system (L).

Little's law:

$$L = \lambda W$$

In this subsection we will validate Little's law using empirical results on Customers tele-queue.

Now we compare the values of λW with number of customers in queue for different types. We will observe that in some cases the two patterns coincide and in other cases there can be large difference between graphs.

Figure 159 - Figure 161 demonstrate the results of our validation for Retail calls in May 2002. We observe that λW and L almost coincide (see Figure 161).

Figure 162 - Figure 164 presents validation results for Retail calls on August 16, 2001. Note that λW and L almost coincide (see Figure 164).

Figure 165 - Figure 167 presents validation results for Telesales calls on October 10, 2001. We observe that the difference between two curves reaches 20 – 25% on 10am 1 pm (see Figure 167).

Figure 168 - Figure 170 demonstrate validation results for Consumer Loans calls on October 10, 2001. Note that difference between two curves reaches 25% on 11:30am: number of customers in queue is 28 and λW is equal to 21 (see Figure 170). It can be connected with the peak of waiting time (and number of customers in queue, respectively) during 10 - 10:30am time period (see Figure 15 in Subsection 3.3).

Little's Law for Retail calls, May 2002

Figure 159 : λ: Throughput Rate, Retail (May-02)

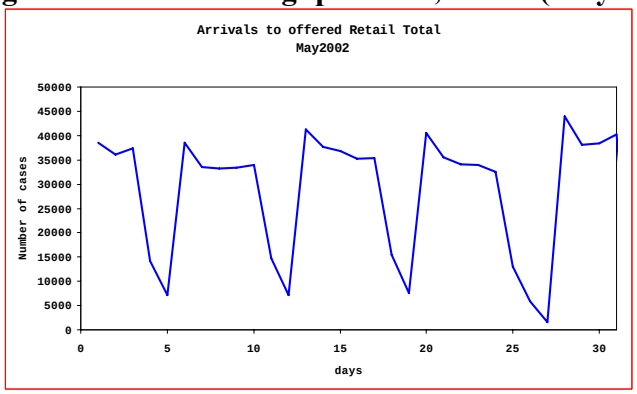


Figure 160 : W: Average Waiting Time, Retail (May-02)

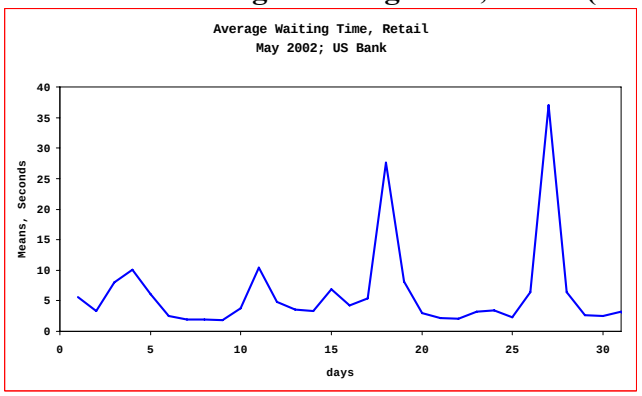
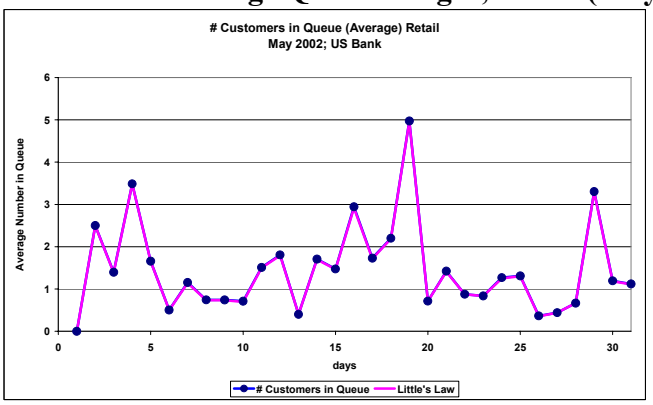


Figure 161 : L: Average Queue Length, Retail (May-02)



Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
λ	38476	36144	37414	14194	7107	38587	33572	33220	33349	34009	14807	7141	41293	37653	36872	35266
W	5.6	3.3	8.0	10.0	6.1	2.6	1.9	1.9	1.8	3.8	10.5	4.8	3.5	3.4	6.8	4.2
λ * W	2.49	1.39	3.48	1.65	0.50	1.14	0.74	0.74	0.69	1.50	1.79	0.40	1.69	1.47	2.91	1.71
L	2.50	1.40	3.49	1.66	0.50	1.15	0.74	0.74	0.71	1.51	1.81	0.40	1.71	1.47	2.94	1.73

Date	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
λ	35338	15533	7530	40534	35493	34070	34005	32512	13100	5909	1558	43980	38163	38416	40284
W	5.4	27.6	8.1	3.0	2.1	2.1	3.2	3.5	2.3	6.4	37.0	6.5	2.7	2.5	3.2
λ * W	2.19	4.96	0.71	1.41	0.87	0.83	1.25	1.30	0.35	0.44	0.67	3.29	1.18	1.11	1.47
L	2.20	4.97	0.71	1.42	0.88	0.84	1.27	1.31	0.37	0.44	0.67	3.30	1.20	1.12	1.48

Little's Law for Retail calls, August 16, 2001

Figure 162 : λ: Throughput Rate, Retail (Thursday, August 16, 2001)

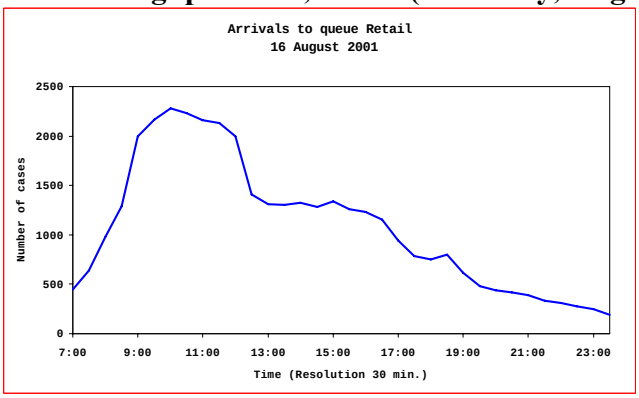


Figure 163 : W: Average Waiting Time, Retail (Thursday, August 16, 2001)

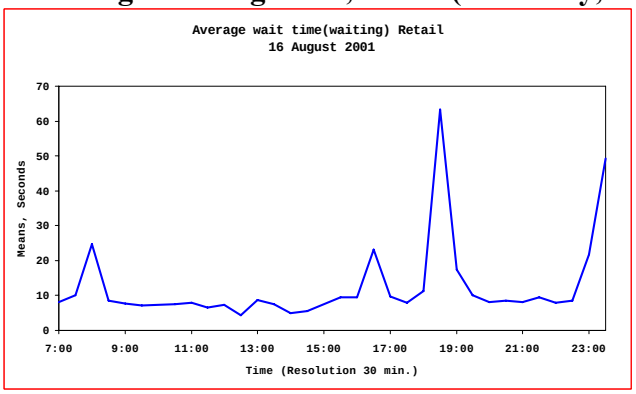
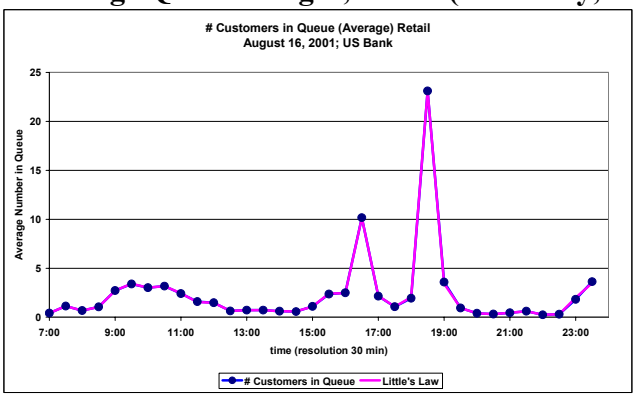


Figure 164 : L: Average Queue Length, Retail (Thursday, August 16, 2001)



Time	7:00	7:30	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:00
λ	443	639	987	1291	1998	2166	2278	2231	2158	2135	2000	1408	1311	1303	1323	1285	1340
W	1.7	3.2	1.2	1.5	2.4	2.8	2.4	2.6	2.0	1.3	1.3	0.8	1.0	1.0	0.8	0.8	1.5
λ * W	0.42	1.14	0.68	1.06	2.72	3.42	3.01	3.18	2.44	1.55	1.47	0.64	0.72	0.72	0.62	0.59	1.09
L	0.42	1.14	0.68	1.06	2.72	3.40	3.02	3.17	2.41	1.59	1.48	0.64	0.72	0.72	0.62	0.57	1.11

Time	15:30	16:00	16:30	17:00	17:30	18:00	18:30	19:00	19:30	20:00	20:30	21:00	21:30	22:00	22:30	23:00	23:30
λ	1258	1235	1157	942	788	752	803	619	485	437	421	386	336	311	274	251	193
W	3.5	3.6	15.8	4.2	2.4	4.9	51.9	10.0	3.5	1.7	1.3	2.1	3.3	1.4	2.0	14.3	32.6
λ * W	2.422	2.45	10.2	2.173	1.06	2.05	23.16	3.43	0.95	0.41	0.314	0.44	0.62	0.24	0.30	2.00	3.50
L	2.37	2.49	10.17	2.16	1.07	1.94	23.11	3.59	0.95	0.40	0.31	0.45	0.62	0.24	0.30	1.83	3.63

Little's Law for Telesales calls, October 10, 2001

Figure 165 : λ: Throughput Rate, Telesales (Wednesday, October 10, 2001)

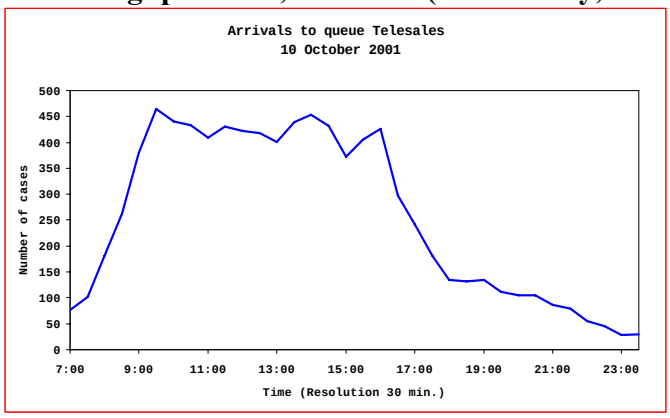


Figure 166 : W: Average Waiting Time, Telesales (Wednesday, Oct 10, 2001)

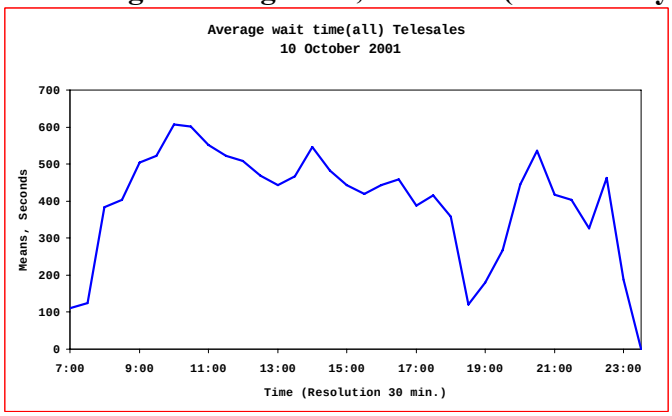
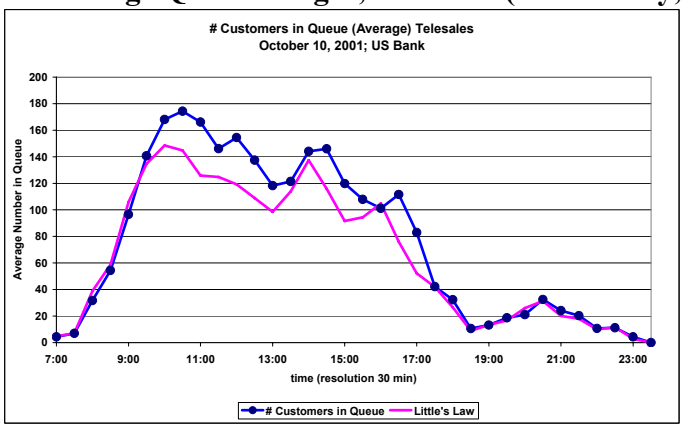


Figure 167 : L: Average Queue Length, Telesales (Wednesday, Oct 10, 2001)



Time	7:00	7:30	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:00
λ	76	102	182	262	379	464	440	433	410	431	422	418	401	439	453	432	373
W	109.8	123.8	383.5	403.7	503.5	522.5	607.9	602.1	552.4	521.1	508.6	468.8	442.1	467.3	545.9	483.1	442.1
λ * W	4.63	7.01	38.77	58.76	106.01	134.69	148.60	144.84	125.82	124.77	119.23	108.86	98.48	113.98	137.39	115.93	91.61
L	4.28	6.91	31.73	54.36	96.50	140.70	168.10	174.34	166.14	146.13	154.48	137.47	118.29	121.44	144.07	146.01	119.83

Time	15:30	16:00	16:30	17:00	17:30	18:00	18:30	19:00	19:30	20:00	20:30	21:00	21:30	22:00	22:30	23:00	23:30
λ	405	427	298	242	182	134	132	134	112	105	105	87	80	55	45	28	30
W	419.2	442.2	458.8	387.9	415.1	357.1	121.6	179.8	267.9	445.7	536.0	416.9	403.9	326.0	463.6	187.3	0.9
λ * W	94.31	104.89	75.96	52.15	41.97	26.58	8.92	13.38	16.67	26.00	31.27	20.15	17.95	9.96	11.59	2.91	0.02
L	107.86	101.22	111.60	82.93	42.23	32.32	10.57	13.24	18.67	21.07	32.50	24.10	20.33	10.69	11.13	4.35	0.02

Little's Law for Consumer Loans calls, October 10, 2001

Figure 168 : λ: Throughput Rate, Consumer Loans (Wednesday, Oct 10, 2001)

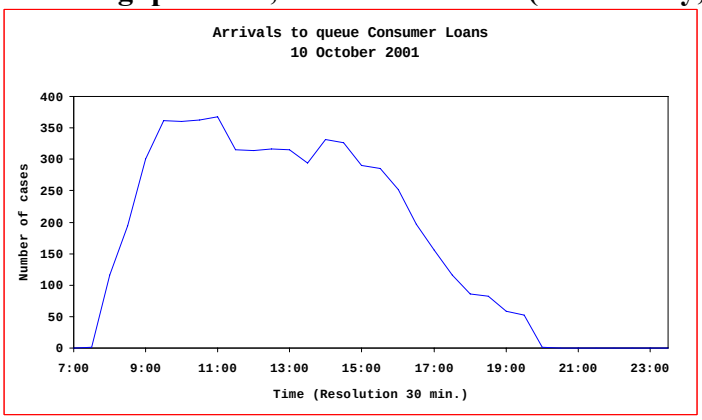


Figure 169 : W: Average Waiting Time, Cons Loans (Wednesday, Oct 10, 2001)

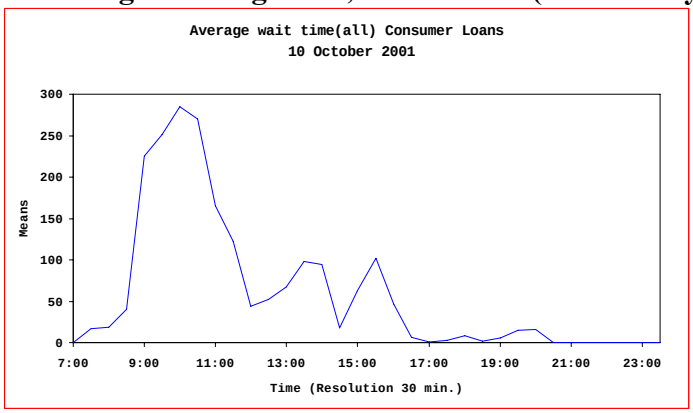
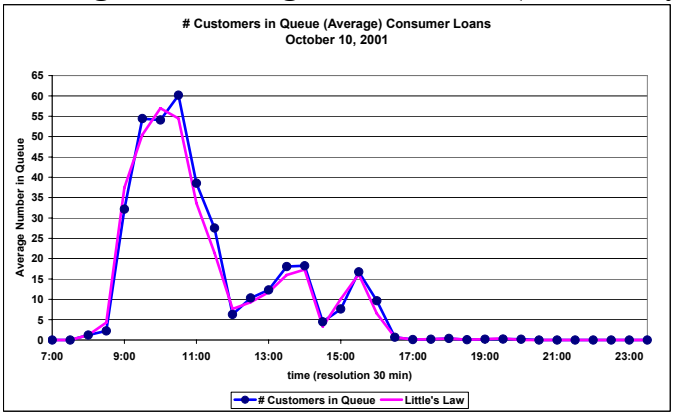


Figure 170 : L: Average Queue Length, Cons Loans (Wednesday, Oct 10, 2001)



Time	7:00	7:30	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:00
λ	0	1	116	194	300	361	360	363	367	315	314	317	315	294	332	326	290
W	0.0	17.0	19.0	40.0	224.8	251.4	284.9	270.0	165.0	122.5	43.8	52.8	67.1	98.0	94.0	18.2	62.4
λ*W	0.00	0.01	1.22	4.31	37.47	50.43	55.98	54.44	33.63	21.44	7.64	9.29	11.75	16.01	17.33	3.30	10.05
L	0.00	0.01	1.22	2.25	32.15	54.42	54.07	60.18	38.53	27.52	6.28	10.33	12.31	18.07	18.24	4.48	7.63

Time	15:30	16:00	16:30	17:00	17:30	18:00	18:30	19:00	19:30	20:00	20:30	21:00	21:30	22:00	22:30	23:00	23:30
λ	288	252	197	156	116	86	82	58	52	1	0	0	0	0	0	0	0
W	102.2	46.4	6.1	1.2	2.8	8.0	2.0	6.0	14.5	16.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
λ*W	16.18	6.49	0.67	0.11	0.18	0.38	0.09	0.19	0.42	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L	16.77	9.67	0.67	0.11	0.18	0.38	0.07	0.21	0.25	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00

References

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Journal of the American Statistical Association, Vol 100: 36-50, 2005

[3] The Call Center of a IL Telecom